Relations and Functions

Question1

The function f : N− {1} → N ; defined by f(n) = the highest prime factor of n, is :

[27-Jan-2024 Shift 1]

Options:

A.

both one-one and onto

B.

one-one only

C.

onto only

D.

neither one-one nor onto

Answer: D

Solution:

$f : N - \{1\} \longrightarrow N$

$f(n) =$ The highest prime factor of n.

$f(2) = 2$

$f(4) = 2$

$\Rightarrow$ many one

4 is not image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

Question2

Let $f : R - \left\{\frac{-1}{2}\right\} \rightarrow R$ and $g : R - \left\{\frac{-5}{2}\right\} \rightarrow R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then the domain of the function fog is :

[27-Jan-2024 Shift 2]

Options:

A.

$R-\left\{-\frac{5}{2}\right\}$

B.

R

C.

$R-\left\{-\frac{7}{4}\right\}$

D.

$R-\left\{-\frac{5}{2},-\frac{7}{4}\right\}$

Answer: A

Solution:

$f(x)=\frac{2x+3}{2x+1}; x\neq-\frac{1}{2}$

$g(x)=\frac{|x|+1}{2x+5}, x\neq-\frac{5}{2}$

Domain of $f(g(x))$

$f(g(x))=\frac{2g(x)+3}{2g(x)+1}$

$x\neq-\frac{5}{2}$ and $\frac{|x|+1}{2x+5}\neq-\frac{1}{2}$

$x\in R-\left\{-\frac{5}{2}\right\}$ and $x\in R$

$\therefore$ Domain will be $R-\left\{-\frac{5}{2}\right\}$

Question3

Consider the function f : [1/2, 1] → R defined by f(x) = 4√2x 3^{-3}√2x^{-1}. Consider the statements

(I) The curve y = f(x) intersects the x-axis exactly at one point

(II) The curve y = f(x) intersects the x-axis at x = cos π/12

Then

[29-Jan-2024 Shift 1]

Options:

A.

Only (II) is correct

B.

Both (I) and (II) are incorrect

C.

Only (I) is correct

D.

Both (I) and (II) are correct

Answer: D

Solution:

$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \ge 0$ for $\left[\frac{1}{2}, 1\right]$

$f\left(\frac{1}{2}\right) < 0$

$f(1) > 0 \Rightarrow$ (A) is correct.

$f(x) = \sqrt{2}(4x^3 - 3x) - 1 = 0$

Let $\cos\alpha = x$,

$\cos 3\alpha = \cos\frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$

$x = \cos\frac{\pi}{12}$

(4) is correct.

Question4

If $f(x) = \begin{cases} 2+2x & -1 \le x < 0 \\ 1 - \frac{x}{3} & 0 \le x \le 3 \end{cases}$

$g(x) = \begin{cases} -x & -3 \le x \le 0 \\ x & 0 < x \le 1 \end{cases}$ ..,

then range of $(fog(x))$ is

[29-Jan-2024 Shift 1]

Options:

A.

(0, 1]

B.

[0, 3)

C.

[0, 1]

D.

[0, 1)

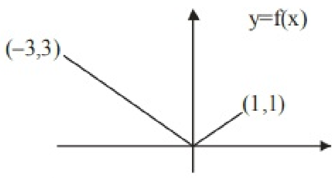
Answer: C

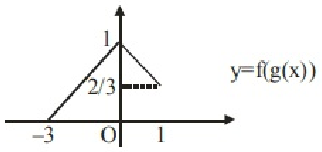
Solution:

$$f(g(x)) = \begin{cases} 2+2g(x), & -1 \le g(x) < 0 \ldots\ldots(1) \\ 1-\frac{g(x)}{3}, & 0 \le g(x) \le 3 \ldots\ldots(2) \end{cases}$$

By (1) $x \in \phi$

And by (2) $x \in [-3, 0]$ and $x \in [0, 1]$





Range of $f(g(x))$ is [0, 1]

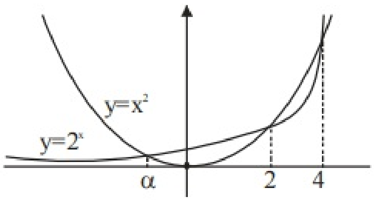
Question5

Let $f(x) = 2^x - x^2$, $x \in R$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersects the x-axis, then the value of m + n is

[29-Jan-2024 Shift 1]

Answer: 5

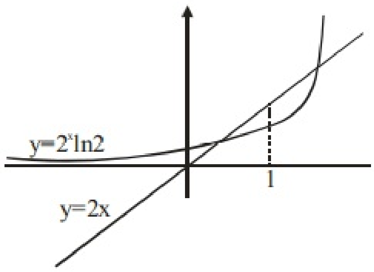
Solution:



$\therefore m = 3$

$f'(x) = 2^x \ln 2 - 2x = 0$

$2^x \ln 2 = 2x$



$\therefore n = 2$

$\Rightarrow m + n = 5$

Question6

If the domain of the function $f(x) = \cos^{-1}(2 - |x|/4) + (\log_e(3 - x))^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

12

B.

9

C.

11

D.

8

Answer: C

Solution:

$-1 \leq \left|\frac{2-|x|}{4}\right| \leq 1$

$\Rightarrow \left|\frac{2-|x|}{4}\right| \leq 1$

$-4 \leq 2-|x| \leq 4$

$-6 \leq -|x| \leq 2$

$-2 \leq |x| \leq 6$

$|x| \leq 6$

$\Rightarrow x \in [-6, 6]$ ....(1)

Now, $3-x \neq 1$

And $x \neq 2$ ....(2)

and $3-x > 0$

$x < 3$ ....(3)

From (1), (2) and (3)

$\Rightarrow x \in [-6, 3) - \{2\}$

$\alpha = 6$

$\beta = 3$

$\gamma = 2$

$\alpha + \beta + \gamma = 11$

Question7

Let A = {1, 2, 3,....7} and let P(1) denote the power set of A. If the number of functions f : A → P(A) such that a ∈ f(a), ∀ a ∈ A is mn,m and n∈N and m is least, then m + n is equal to______

[30-Jan-2024 Shift 1]

Answer: 44

Solution:

$f : A \longrightarrow P(A)$

$a \in f(a)$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6 . (Because 2^6 subsets contains 1)

Similarly, for every other element

Hence, total is $2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 = 2^{42}$

Ans. $2 + 42 = 44$

Question8

If the domain of the function f(x) = $\log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is (α, β], then the value of 5β − 4α is equal to

[30-Jan-2024 Shift 2]

Options:

A.

10

B.

12

C.

11

D.

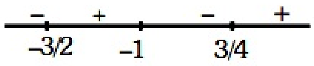
9

Answer: B

Solution:

$\frac{2x+3}{4x^2+x-3}>0$ and $-1\leq\frac{2x-1}{x+2}\leq1$

$\frac{2\times+3}{(4x-3)(x+1)}>0$ $\frac{3x+1}{x+2}\geq0$ & $\frac{x-3}{x+2}\leq0$



$(-\infty,-2)\cup\left[\frac{-1}{3},\infty\right)$ ...... (1)

$(-2,3]$ .... (2)

$\left[\frac{-1}{3},3\right]$..... (3) (1) ∩ (2) ∩ (3)

$\left(\frac{3}{4},3\right]$

$\alpha=\frac{3}{4}\beta=3$

$5\beta-4\alpha=15-3=12$

Question9

Let f : R → R be a function defined f(x) = $\frac{x}{(1+x^4)^{1/4}}$ and g(x) = f(f(f(f(x)))) then 18 $\int_0^{\sqrt{2\sqrt{5}}}x^2g(x)\,dx$

[30-Jan-2024 Shift 2]

Options:

A.

33

B.

36

C.

42

D.

39

Answer: D

Solution:

$f(x)=\frac{x}{(1+x^4)^{1/4}}$

$fof(x)=\frac{f(x)}{(1+f(x)^4)^{1/4}}=\frac{\frac{x}{(1+x^4)^{1/4}}}{\left(1+\frac{x^4}{1+x^4}\right)^{1/4}}=\frac{x}{(1+2x^4)^{1/4}}$

$f(f(f(f(x))))=\frac{x}{(1+4x^4)^{1/4}}$

$18\int_0^{\sqrt{2\sqrt{5}}}\frac{x^3}{(1+4x^4)^{1/4}}dx$

Let $1+4x^4=t^4$

$16x^3\,dx=4t^3\,dt$

$\frac{18}{4}\int_1^3\frac{t^3\,dt}{t}$

$=\frac{9}{2}\left(\frac{t^3}{3}\right)_1^3$

$=\frac{3}{2}[26]=39$

Question10

If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$ and $(fof)(x)=g(x)$, where $g:\mathbb{R}-\left\{\frac{2}{3}\right\}\to\mathbb{R}-\left\{\frac{2}{3}\right\}$, then (gogog) (4) is equal to

[31-Jan-2024 Shift 1]

Options:

A.

$-\frac{19}{20}$

B.

19/20

C.

-4

D.

4

Answer: D

Solution:

$$f(x) = \frac{4x+3}{6x-4}$$

$$g(x) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{34x}{34} = x$$

$$g(x) = x \therefore g(g(g(4))) = 4$$

Question11

If the function f : (−∞, −1] ⟶ (a, b] defined by f(x) = e^{x^3-3x+1} is one-one and onto, then the distance of the point

P(2b + 4, a+ 2) from the line x + e^{-3}y = 4 is :

[31-Jan-2024 Shift 2]

Options:

A.

$2\sqrt{1+e^6}$

B.

$4\sqrt{1+e^6}$

C.

$3\sqrt{1+e^6}$

D.

$\sqrt{1+e^6}$

Answer: A

Solution:

$f(x) = e^{x^3 - 3x + 1}$

$f'(x) = e^{x^3 - 3x + 1} \cdot (3x^2 - 3)$

$= e^{x^3 - 3x + 1} \cdot 3(x - 1)(x + 1)$

For $f'(x) \geq 0$

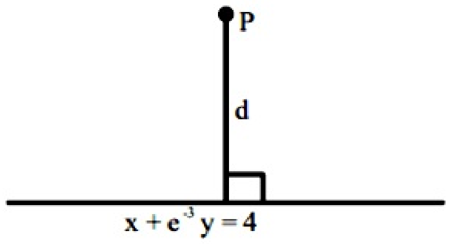
$\therefore f(x)$ is increasing function

$\therefore a = e^{-\infty} = 0 = f(-\infty)$

$b = e^{-1+3+1} = e^3 = f(-1)$

$P(2b + 4, a + 2)$

$\therefore P(2e^3 + 4, 2)$



$$d = \frac{(2e^3 + 4) + 2e^{-3} - 4}{\sqrt{1 + e^{-6}}} = 2\sqrt{1 + e^6}$$

Question12

Let $f : R \longrightarrow R$ and $g : R \longrightarrow R$ be defined as

$$f(x) = \begin{cases} \log_e x & , \quad x > 0 \\ e^{-x} & , \quad x \leq 0 \end{cases}.$$

and

$$g(x) = \begin{cases} x & , \quad x \geq 0 \\ e^x & , \quad x < 0 \end{cases}.$$

Then, gof : $R \longrightarrow R$ is :

[1-Feb-2024 Shift 1]

Options:

A.

one-one but not onto

B.

neither one-one nor onto

C.

onto but not one-one

D.

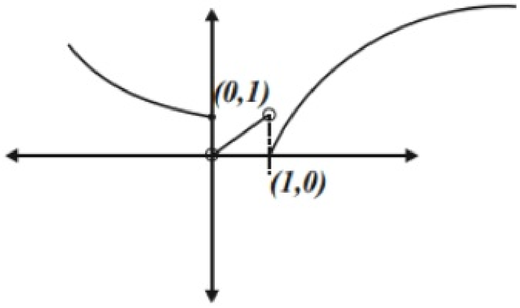
both one-one and onto

Answer: B

Solution:

$$g(f(x)) = \begin{cases} f(x) & f(x) \geq 0 \\ e^{f(x)} & f(x) < 0 \end{cases}.$$

$$g(f(x)) = \begin{cases} e^{-x} & (-\infty, 0] \\ e^{\ln x} & (0, 1) \\ \ln x & [1, \infty) \end{cases}.$$



Graph of $g(f(x))$

$g(f(x)) \Rightarrow$ Many one into

Question13

If the domain of the function f(x) = $\frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$ is $(-\infty, \alpha) U [\beta, \infty)$, **then $\alpha^2 + \beta^3$ is equal to :**

[1-Feb-2024 Shift 2]

Options:

A.

140

B.

175

C.

150

D.

Answer: C

Solution:

$$f(x) = \frac{\sqrt{x^2-25}}{4-x^2} + \log_{10}(x^2+2x-15)$$

Domain $: x^2 - 25 \geq 0 \Rightarrow x \in (-\infty, -5] \cup [5, \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2, 2\}$

$x^2 + 2x - 15 > 0 \Rightarrow (x+5)(x-3) > 0$

$\Rightarrow x \in (-\infty, -5) \cup (3, \infty)$

$\therefore x \in (-\infty, -5) \cup [5, \infty)$

$\alpha = -5; \beta = 5$

$\therefore \alpha^2 + \beta^3 = 150$

Question14

Let A = {1, 2, 3, 4, ..., 10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation $R = \{ (a, b) \in A \times A: 2(a-b)^2 + 3(a-b) \in B \}$ is ______.

[6-Apr-2023 shift 1]

Answer: 18

Solution:

Solution:

$A = \{1, 2, 3, \ldots\ldots 10\}$

$B = \{0, 1, 2, 3, 4\}$

$R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$

Now $2(a-b)^2 + 3(a-b) = (a-b)(2(a-b)+3)$

$\Rightarrow a = b$ or $a - b = -2$

When $a = b \Rightarrow 10$ order pairs

When $a - b = -2 \Rightarrow 8$ order pairs

Total $= 18$

Question15

Let A = {0, 34, 6, 7, 8, 9, 10} and R be the relation defined on A such that $R = \{x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be aadded to the relation R, so that it is a symmetric relation, is equal to ______.

[8-Apr-2023 shift 1]

Answer: 19

Solution:

Solution:
$A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ $3, 7, 9 \rightarrow$ odd
$R = \{x - y =$ odd $+$ ve or $x - y = 2\}$ $0, 4, 6, 8, 10 \rightarrow$ even
${}^3C_1 \cdot {}^5C_1 = 15 + (6, 4), (8, 6), (10, 8), (9, 7)$
Min ^{m} ordered pairs to be added must be
$: 15 + 4 = 19$

Question16

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is
[8-Apr-2023 shift 2]

Options:

A. Symmetric but neither reflexive nor transitive

B. Transitive but neither symmetric nor reflexive

C. An equivalence relation

D. Reflexive but neither symmetric nor transitive

Answer: A

Solution:

Solution:
$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

Question17

Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times . B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is
[10-Apr-2023 shift 2]

Options:

A. 18

B. 24

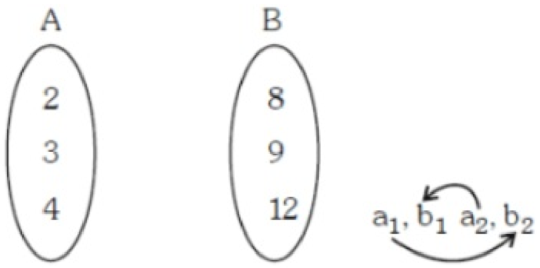
C. 12

D. 36

Answer: D

Solution:

Solution:



a_1 divides b_2
Each elements has 2 choices
$\Rightarrow 3 \times 2 = 6$
a_2 divides b_1
Each elements has 2 choices
$\Rightarrow 3 \times 2 = 6$
Total $= 6 \times 6 = 36$

Question18

Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that R = { ((a_1, b_1), (a_2, b_2..)): $a_1 \le b_2$ and $b_1 \le a_2$ }. Then the number of elements in the set R is
[11-Apr-2023 shift 2]

Options:

A. 52

B. 160

C. 26

D. 180

Answer: B

Solution:

Solution:
Let $a_1 = 1 \Rightarrow 5$ choices of b_2
$a_1 = 3 \Rightarrow 4$ choices of b_2
$a_1 = 4 \Rightarrow 4$ choices of b_2
$a_1 = 6 \Rightarrow 2$ choices of b_2
$a_1 = 9 \Rightarrow 1$ choices of b_2
For (a_1, b_2)16 ways .
Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2
$b_1 = 4 \Rightarrow 3$ choices of a_2
$b_1 = 5 \Rightarrow 2$ choices of a_2

$b_1 = 8 \Rightarrow 1$ choices of a_2
Required elements in $R = 160$

Question19

The number of the relations, on the set {1, 2, 3} containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is ________.
[12-Apr-2023 shift 1]

Answer: 3

Solution:

Solution:
$A = \{1, 2, 3\}$
For Reflexive $(1, 1)(2, 2), (3, 3) \in R$
For transitive : $(1, 2)$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$
Not symmetric : $(2, 1)$ and $(3, 2) \notin R$
$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$
$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$
$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

Question20

Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{ (a, b) \in A \times A : b = |a|$. or ^{2}=a + 1 } be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is ________
[13-Apr-2023 shift 2]

Answer: 7

Solution:

Solution:
$R = [(-4, 4), (-3, 3), (3, -2), (0, 1), (0, 0), (1, 1),$
$(4, 4), (3, 3) \}$
For reflexive, add $\Rightarrow (-2, -2), (-4, -4), (-3, -3)$
For symmetric, add $\Rightarrow (4, -4), (3, -3), (-2, 3), (1, 0)$

Question21

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by $R = \{ ((a, b, (c, d) : 2a + 3b = 4c + 5d \}$. Then the number of elements in R is ________
[15-Apr-2023 shift 1]

Answer: 6

Solution:

Solution:
$A = \{1, 2, 3, 4\}$
$R = \{(a, b), (c, d)\}$
$2a + 3b = 4c + 5d = \alpha$ let
$2a = \{2, 4, 6, 8\}$ $4c = \{4, 8, 12, 16\}$
$3b = \{3, 6, 9, 12\}$ $5d = \{5, 10, 15, 20\}$

$$2a + 3b = \left\{ \begin{matrix} 5 & 8 & 11 & 14 \\ 7 & 10 & 13 & 16 \\ 9 & 12 & 15 & 18 \\ 11 & 14 & 17 & 20 \end{matrix} \right\} \quad 4c + 5d \left\{ \begin{matrix} 9 & 14 & 19 & 24 \\ 13 & 18\ldots & & \\ 17 & 22\ldots. & & \\ 21 & 26\ldots. & & \end{matrix} \right\}$$

Possible value of $\alpha = 9, 13, 14, 14, 17, 18$
Pairs of $\{(a, b), (c, d)\} = 6$

Question22

Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18\int_1^2 f(x)\,dx$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. $10\log_e 2 - 6$

B. $10\log_e 2 + 6$

C. $5\log_e 2 - 3$

D. $5\log_e 2 + 3$

Answer: A

Solution:

Solution:
$5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \ldots (1)$
$x \to \frac{1}{x}$
$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3 \ldots (2)$
$(1) \times 5 - (2) \times 4$

$\Rightarrow f(x) = \frac{5}{9x} - \frac{4}{9}x + \frac{1}{3}$

$\Rightarrow 18\int_1^2 f(x)\,dx = 18\left(\frac{5}{9}\ln 2 - \frac{4}{9}\times\frac{3}{2} + \frac{1}{3}\right)$

$= 10\ln 2 - 6$

Question23

Let A = $\{x \in \mathbb{R} : [x+3] + [x+4] \le 3\}$,

B = $\left\{ x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^x} \right)^{x-3} < 3^{-3x} \right\}$, where [t] denotes greatest integer function. Then,

[6-Apr-2023 shift 1]

Options:

A. $A \subset B, A \neq B$

B. $A \cap B = \varphi$

C. $A = B$

D. $B \subset C, A \neq B$

Answer: C

Solution:

Solution:

$A = \{x \in \mathbb{R} : [x+3] + [x+4] \le 3\}$

$2[x] + 7 \le 3$

$2[x] \le -4$

$[x] \le -2 \Rightarrow x < -1 \ldots (A)$

$B = \left\{ x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^x} \right)^{x-3} < 3^{-3x} \right\}$

$3^x \left(\sum_{r-1}^{\infty} \frac{3}{10^x} \right)^{x-3} < 3^{-3x}$

$3^{2x-3}\left(\frac{10}{10}\right)^{x-3}$

$\Rightarrow \left(\frac{1}{9}\right)^{x-3} < 3^{-5x}$

$\Rightarrow 3^{6-2x} < 3^{3-5x}$

$\Rightarrow 6 - 2x < 3 - 5x$

$\Rightarrow 3 < -3x$

$\Rightarrow \frac{1}{10})^x < -1 \ldots (B)$

$A = B$

Question24

Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{[x] - x}}$, where [x] denotes the smallest integer greater than or equal to x. Then among the statements :

(S1) : $A \cap B = (1, \infty) - N$ and

(S2) : A ∪ B = (1, ∞)
[6-Apr-2023 shift 2]

Options:

A. only (S1) is true

B. neither (S1) nor (S2) is true

C. only (S2) is true

D. both (S1) and (S2) are true

Answer: A

Solution:

Solution:

$f(x) = \frac{1}{\sqrt{[x] - x}}$

If $x \in I$ $[x] = [x]$ (greatest integer function)

If $x \notin I[x] = [x] + 1$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{[x] - x}} & x \in I \\ \frac{1}{\sqrt{[x] + 1 - x}} & x \notin I \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{-\{x\}}} & x \in I \quad (\text{ does not exist }) \\ \frac{1}{\sqrt{1 - \{x\}}} & x \notin I \end{cases}.$$

$\Rightarrow$ domain of $f(x) = R - I$

Now, $f(x) = \frac{1}{\sqrt{1 - \{x\}}}, x \notin I$

$\Rightarrow x < \{x\} < 1$

$\Rightarrow 0 < 1\sqrt{1 - \{x\}} < 1$

$\Rightarrow \frac{1}{\sqrt{1 - \{x\}}} > 1$

$\Rightarrow$ Range$(1, \infty)$

$\Rightarrow A = R - I$

$B = (1 \infty)$

So, $A \cap B = (1, \infty) - N$

$A \cup B \neq (1, \infty)$

$\Rightarrow$ S1 is only correct.

Question25

Let f, g : ℕ − {1} → ℕ be functions defined by f(a) = α, where α is the maximum of the powers of those primes p such that p^{α} divides a, and g(a) = a + 1, for all a ∈ ℕ − {1}. Then, the function f + g is [27-Jul-2022-Shift-1]

Options:

A. one-one but not onto

B. onto but not one-one

C. both one-one and onto

D. neither one-one nor onto

Answer: D

Solution:

Solution:
$f, g : N - \{1\} \rightarrow N$ defined as
$f(a) = \alpha$, where α is the maximum power of those primes p such that p^{α} divides a.
$g(a) = a + 1$
Now,
$f(2) = 1, \quad g(2) = 3 \quad \Rightarrow \quad (f + g)(2) = 4$
$f(3) = 1, \quad g(3) = 4 \quad \Rightarrow \quad (f + g)(3) = 5$
$f(4) = 2, \quad g(4) = 5 \Rightarrow (f + g)(4) = 7$
$f(5) = 1, \quad g(5) = 6 \Rightarrow (f + g)(5) = 7$
$\because (f + g)(5) = (f + g)(4)$
$\therefore f + g$ is not one-one
Now, $\because f_{min} = 1, g_{min} = 3$
So, there does not exist any $x \in N - \{1\}$ such that $(f + g)(x) = 1, 2, 3$
$\therefore f + g$ is not onto

Question26

If domain of the function $\log_e\left(\frac{6x^2 + 5x + 1}{2x - 1}\right) + \cos^{-1}\left(\frac{2x^2 - 3x + 4}{3x - 5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then, $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to [8-Apr-2023 shift 2]

Answer: 20

Solution:

Solution:

$$\frac{6x^2 + 5x + 1}{2x - 1} > 0$$

$$\frac{(3x + 1)(2x + 1)}{2x - 1} > 0$$

$$x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \cup \left(\frac{5}{3}, \infty\right) \ldots (B)$$

$$x < \frac{5}{3} \ldots (C)$$

$$A \cap B \cap C \equiv \left(\frac{-1}{2}, \frac{-1}{3}\right) \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$$

$$\text{So } 18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) = 18\left(\frac{1}{4} + \frac{1}{9} + \frac{1}{4} + \frac{1}{2}\right)$$

$$= 18 + 2 = 20$$

Question27

Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto functions f : R → S such that f(a) ≠ 1, is equal to ______.
[8-Apr-2023 shift 2]

Answer: 180

Solution:

Solution:
Total onto function
$\frac{5}{\lfloor 3 \lfloor} \times \lfloor 4 = 240$
Now when f (a) = 1
$\lfloor 4 + \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \lfloor x3 = 24 + 36 = 60.$
so required $f^n = 240 - 60 = 180$

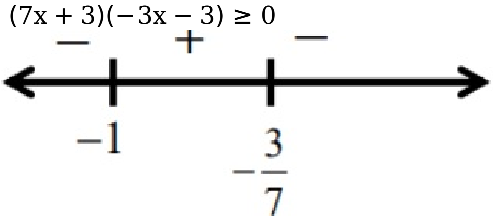
Question28

If the domain of the function f(x) = $\sec^{-1}\left(\frac{2x}{5x+3}\right)$ is [α, β)U(γ, δ], then |3α + 10(β + γ) + 21δ| is equal to _______.
[10-Apr-2023 shift 2]

Answer: 24

Solution:

Solution:
$f(x) = \sec^{-1}\frac{2x}{5x+3}$
$\left|\frac{2x}{5x+3}\right|$
$\left|\frac{2x}{5x+3}\right| \geq 1 \Rightarrow \left|2x\right| \geq \left|5x+3\right|$
$(2x)^2 - (5x+3)^2 \geq 0$
$(7x+3)(-3x-3) \geq 0$



$\therefore$ domain $\left[-1, \frac{-3}{5}\right) \cup \left(\frac{-3}{5}, \frac{-3}{7}\right]$
$\alpha = -1, \beta = \frac{-3}{5}, \gamma = \frac{-3}{5}, \delta = \frac{-3}{7}$
$3\alpha + 10(\beta + \gamma) + 21\delta = -3$
$-3 + 10\left(\frac{-6}{5}\right) + \left(\frac{-3}{7}\right)21 = -24$

Question29

The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is (where [x] denotes the greatest integer less than or equal to x)
[11-Apr-2023 shift 2]

Options:

A. $(-\infty, -3] \cup [6, \infty)$

B. $(-\infty, -2) \cup (5, \infty)$

C. $(-\infty, -3] \cup (5, \infty)$

D. $(-\infty, -2) \cup [6, \infty)$

Answer: D

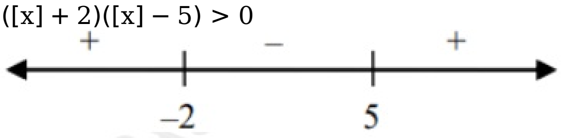
Solution:

Solution:

$F(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$

$[x]^2 - 3[x] - 10 > 0$

$([x] + 2)([x] - 5) > 0$



$[x] < -2$ or $[x] > 5$

$[x] \leq -3$ or $[x] \geq 6$

$x < -2$ or $x \geq 6$

$x \in (-\infty, -2) \cup [6, \infty)$

Question30

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Then the number of functions $f: A \rightarrow B$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to ________.
[11-Apr-2023 shift 2]

Answer: 360

Solution:

Solution:

$f(1) + f(2) + 1 = f(4) \leq 6$

$f(1) + f(2) \leq 5$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow 4$ mappings

Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow 3$ mappings

Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow 2$ mappings

Case (iv) $f(1)4 \Rightarrow f(2) = 1 \Rightarrow 1$ mapping

f (5)&f(6) both have 6 mappings each
Number of functions $= (4+3+2+1) \times 6 \times 6 = 360$

Question31

Let D be the domain of the function $f(x) = \sin^{-1}\left(\log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right)\right)$. If the range of the function $g: D \rightarrow R$ defined by $g(x) = x - [x]$, ($[x]$ is the greatest integer function), is (α, β), then $\alpha^2 + \frac{5}{\beta}$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 46

B. 135

C. 136

D. 45

Answer: B

Solution:

Solution:

$\frac{6+2\log_3 x}{-5x} > 0 \& x > 0 \& x \neq \frac{1}{3}$

this gives $x \in \left(0, \frac{1}{27}\right) \ldots (1)$

$-1 \leq \log_{3x}\left(\frac{6+2\log_3 x}{-5x}\right) \leq 1$

$3x \leq \frac{6+2\log_3 x}{-5x} \leq \frac{1}{3x}$

$15x^2 + 6 + 2\log_3 x \geq 0 \quad 6 + 2\log_3 x + \frac{5}{3} \geq 0$

$x \in \left(0, \frac{1}{27}\right) \ldots (2) \quad x \geq 3^{-\frac{23}{6}} \ldots . (3)$

form (1), (2) & (3)

$x \in \left[3^{-\frac{23}{6}}, \frac{1}{27}\right)$

$\therefore \alpha$ is small positive quantity

$\& \beta = \frac{1}{27}$

$\therefore \alpha^2 + \frac{5}{\beta}$ is just greater than 135

Question32

For $x \in R$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x} + 1$ and $fog(x) = x + 3 - \sqrt{x}$. Then $f(0)$ is equal to

[13-Apr-2023 shift 1]

Options:

A. 5

B. 0

C. -3

D. 1

Answer: A

Solution:

Solution:

$g(x) = \sqrt{x} + 1$

$fog\ (x) = x + 3 - \sqrt{x}$

$= (\sqrt{x} + 1)^2 - 3(\sqrt{x} + 1) + 5$

$= g^2(x) - 3g(x) + 5$

$\Rightarrow f(x) = x^2 - 3x + 5$

$\therefore f(0) = 5$

But, if we consider the domain of the composite function $fog\ (x)$ then in that case $f(0)$ will be not defined as $g(x)$ cannot be equal to zero.

Question33

For the differentiable function $f : R - \{0\} \to R$, let $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$, then $\left|f(3) + f'\left(\frac{1}{4}\right)\right|$ is equal to
[13-Apr-2023 shift 1]

Options:

A. 13

B. $\frac{29}{5}$

C. $\frac{33}{5}$

D. 7

Answer: A

Solution:

Solution:

$\left[3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10\right] \times 3$

$\left[2f(x) + 3f\left(\frac{1}{x}\right) = x - 10\right] \times 2$

$5f(x) = \frac{3}{x} - 2x - 10$

$f(x) = \frac{1}{5}\left(\frac{3}{x} - 2x - 10\right)$

$f'(x) = \frac{1}{5}\left(-\frac{3}{x^2} - 2\right)$

$\left| f(3) + f'\left(\frac{1}{4}\right) \right| = \left| \frac{1}{5}(1 - 6 - 10) + \frac{1}{5}(-48 - 2) \right|$

$= |-3 - 10| = 13$

Question34

The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

[13-Apr-2023 shift 2]

Options:

A. $[0, \pi)$

B. $[0, \pi]$

C. $[0, 2\pi)$

D. $[0, 2\pi]$

Answer: C

Solution:

Solution:

$f(x) = 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$

$0 \le \frac{x^2}{1+x^2} < 1$

$\Rightarrow 0 \le \sin^{-1}\left(\frac{x^2}{1+x^2}\right) < \frac{\pi}{2}$

$\Rightarrow 0 \le 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right) < 2\pi$

Range : $[0, 2\pi)$

Question35

If the domain of the function

$f(x) = \log_e(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha, \beta]$, then $36|\alpha + \beta|$ is equal to

[15-Apr-2023 shift 1]

Options:

A. 72

B. 63

C. 45

D. 54

Answer: C

Solution:

Solution:

$f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$+ \cos^{-1}\left(\frac{10x + 6}{3}\right)$

(i) $4x^2 + 11x + 6 > 0$

$4x^2 + 8x + 3x + 6 > 0$

$(4x + 3)(x + 2) > 0$

$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$

(ii) $4x + 3 \in [-1, 1]$

$x \in [-1, -1/2]$

(iii) $\frac{10x + 6}{3} \in [-1, 1]$

$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$

$x \in \left(-\frac{3}{4}, -\frac{1}{2}\right]$ $\alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$

$\alpha + \beta = -\frac{5}{4}$

$36 | \alpha + \beta | = 45$

Question36

The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is: [24-Jan-2023 Shift 1]

Options:

A. transitive but not reflexive

B. symmetric but not transitive

C. reflexive but not symmetric

D. neither symmetric nor transitive

Answer: D

Solution:

Solution:

Reflexive : (a, a) gcd of (a, a) = 1

Which is not true for every a E Z.

Symmetric:

Take $a = 2, b = 1$ $\gcd(2, 1) = 1$

Also $2a = 4 \neq b$

Now when $a = 1, b = 2$ $\gcd(1, 2) = 1$

Also now $2a = 2 = b$

Hence $a = 2b$

⇒R is not Symmetric

Transitive:

Let $a = 14, b = 19, c = 21$

$\gcd(a, b) = 1$

$\gcd(b, c) = 1$

$\gcd(a, c) = 7$

Hence not transitive
R is neither symmetric nor transitive.

Question37

Let R be a relation defined on N as a R b is 2a + 3b is a multiple of 5, a, b $\in \mathbb{N}$. Then R is [29-Jan-2023 Shift 2]

Options:

A. not reflexive

B. transitive but not symmetric

C. symmetric but not transitive

D. an equivalence relation

Answer: D

Solution:

Solution:
a Ra ⇒ 5a is multiple it 5
So reflexive
aRb ⇒ $2a + 3b = 5\alpha$,
Now b R a

$2b + 3a = 2b + \left(\frac{5\alpha - 3b}{2}\right) \cdot 3$

$= \frac{15}{2}\alpha - \frac{5}{2}b = \frac{5}{2}(3\alpha - b)$

$= \frac{5}{2}(2a + 2b - 2\alpha)$

$= 5(a + b - \alpha)$

Hence symmetric

a R b $\Rightarrow 2a + 3b = 5\alpha$.

b R c $\Rightarrow 2b + 3c = 5\beta$

Now $2a + 5b + 3c = 5(\alpha + \beta)$

$\Rightarrow 2a + 5b + 3c = 5(\alpha + \beta)$

$\Rightarrow 2a + 3c = 5(\alpha + \beta - b)$

$\Rightarrow$ aRc

Hence relation is equivalence relation.

Question38

The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that it becomes symmetric and transitive is: [30-Jan-2023 Shift 1]

Options:

A. 4

B. 7

C. 5

D. 3

Answer: B

Solution:

Solution:
For Symmetric $(a, b), (b, c) \in R$
$\Rightarrow (b, a), (c, b) \in R$
For Transitive $(a, b), (b, c) \in R$
$\Rightarrow (a, c) \in R$
Now
1. Symmetric
$\therefore (a, c) \in R \Rightarrow (c, a) \in R$
2. Transitive
$\therefore (a, b), (b, a) \in R$
$\Rightarrow (a, a) \in R \& (b, c), (c, b) \in R$
$\Rightarrow (b, b) \& (c, c) \in R$
$\therefore$ Elements to be added
$$\left\{ \begin{matrix} (b, a) & (c, b) & (a, c) & (c, a) \\ ,(a, a) & (b, b) & (c, c) & \end{matrix} \right\}$$
Number of elements to be added $= 7$

Question39

Let R be a relation on N × N defined by $(a, b) R (c, d)$ if and only if $ad(b - c) = bc(a - d)$. Then R is [31-Jan-2023 Shift 1]

Options:

A. symmetric but neither reflexive nor transitive

B. transitive but neither reflexive nor symmetric

C. reflexive and symmetric but not transitive

D. symmetric and transitive but not reflexive

Answer: A

Solution:

Solution:
$(a, b) R (c, d) \Rightarrow ad(b - c) = bc(a - d)$
Symmetric:
$(c, d) R (a, b) \Rightarrow cb(d - a) = da(c - b) \Rightarrow$
Symmetric
Reflexive:
$(a, b) R (a, b) \Rightarrow ab(b - a) \neq ba(a - b) \Rightarrow$
Not reflexive
Transitive: $(2, 3) R (3, 2)$ and $(3, 2) R (5, 30)$ but
$((2, 3), (5, 30)) \notin R \Rightarrow$ Not transitive

Question40

Among the relations

$$S = \left\{ (a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0 \right\}$$

And T = $\{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in Z \}$,
[31-Jan-2023 Shift 2]

Options:

A. S is transitive but T is not

B. T is symmetric but S is not

C. Neither S nor T is transitive

D. Both S and T are symmetric

Answer: B

Solution:

Solution:
For relation $T = a^2 - b^2 = -I$
Then, (b, a) on relation R
$\Rightarrow b^2 - a^2 = -I$
$\therefore$T is symmetric

$$S = \left\{ (a, b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0 \right\}$$

$$2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2, \Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If $(b, a) \in S$ then
$2 + \frac{b}{a}$ not necessarily positive
$\therefore$S is not symmetric

Question41

Let R be a relation on $\mathbb{R}$, given by $R = \{ (a, b) : 3a - 3b + \sqrt{7}$ is an irrational number $\}$. Then R is
[1-Feb-2023 Shift 1]

Options:

A. Reflexive but neither symmetric nor transitive

B. Reflexive and transitive but not symmetric

C. Reflexive and symmetric but not transitive

D. An equivalence relation

Answer: A

Solution:

Solution:
Check for reflexivity:
As 3(a-a) +$\sqrt{7}$ = $\sqrt{7}$ which belongs to relation so relation is reflexive
Check for symmetric:
Take $a = \frac{\sqrt{7}}{3}, b = 0$
Now $(a, b) \in R$ but $(b, a) \notin R$
As $3(b - a) + \sqrt{7} = 0$ which is rational so relation is not symmetric.
Check for Transitivity:
Take (a, b) as $\left(\frac{\sqrt{7}}{3}, 1\right)$
&(b, c) as $\left(1, \frac{2\sqrt{7}}{3}\right)$
So now $(a, b) \in R \& (b, c) \in R$ but $(a, c) \notin R$ which means relation is not transitive

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Question42

Let P(S) denote the power set of S = {1, 2, 3, ..., 10}. Define the relations R_1 and R_2 on P(S) as AR_1B if $(A \cap B^c) \cup (B \cap A^9) = \varnothing$ and AR_2B if $A \cup B^c = B \cup A^c$, $\forall A, B \in P(S)$. Then :
[1-Feb-2023 Shift 2]

Options:

A. both R_1 and R_2 are equivalence relations

B. only R_1 is an equivalence relation

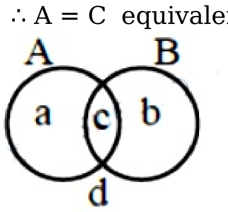
C. only R_2 is an equivalence relation

D. both R_1 and R_2 are not equivalence relations

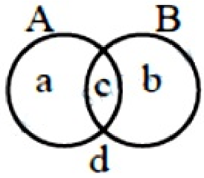
Answer: A

Solution:

Solution:
$S = \{1, 2, 3, \ldots..10\}$
$P(S) =$ power set of S
$AR, B \Rightarrow (A \cap \vec{B}) \cup (\vec{A} \cap B) = \varphi$
R1 is reflexive, symmetric
For transitive
$(A \cap \vec{B}) \cup (\vec{A} \cap B) = \varphi; \{a\} = \varphi = \{b\} A = B$
$(B \cap \vec{C}) \cup (\vec{B} \cap C) = \varphi \therefore B = C$
$\therefore A = C$ equivalence.



$R_2 \equiv A \cup \vec{B} = \vec{A} \cup B$
$R_2 \rightarrow$ Reflexive, symmetric
for transitive



$A \cup \vec{B} = \vec{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$
$\{a\} = \{b\} \therefore A = B$
$B \cup \vec{C} = \vec{B} \cup C \Rightarrow B = C$
$\therefore A = C \quad \therefore A \cup \vec{C} = \vec{A} \cup C \therefore$ Equivalence

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Question43

The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has:
[24-Jan-2023 Shift 1]

Options:

A. exactly two solutions in $(-\infty, \infty)$

B. no solution

C. a unique solution in $(-\infty, 1)$

D. a unique solution in $(-\infty, \infty)$

Answer: D

Solution:

Solution:
$x^2 - 4x + [x] + 3 = x[x]$
$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$
$(x-1)(x-3) = [x].(x-1)$
$\Rightarrow x = 1$ or $x - 3 = [x]$
$\Rightarrow x - [x] = 3$
$\{x\} = 3$ (Not Possible)
Only one solution $x = 1$ in $(-\infty, \infty)$

Question44

Let $f(x)$ be a function such that $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is
[24-Jan-2023 Shift 2]

Options:

A. 6

B. 8

C. 7

D. 9

Answer: C

Solution:

Solution:

$f(x+y) = f(x) \cdot f(y)\ \forall x, y \in N, f(1) = 3$

$f(2) = f^2(1) = 3^2$

$f(3) = f(1)f(2) = 3^3$

$f(4) = 3^4$

$f(k) = 3^k$

$\sum_{k=1}^{n} f(k) = 3279$

$f(1) + f(2) + f(3) + \ldots\ldots\ldots + f(k) = 3279$

$3 + 3^2 + 3^3 + \ldots\ldots\ldots.\ 3^k = 3279$

$\frac{3(3^k - 1)}{3 - 1} = 3279$

$\frac{3^k - 1}{2} = 1093$

$3^k - 1 = 2186$

$3^k = 2187$

$k = 7$

Question45

If $f(x) = \frac{2^{2x}}{2^{2x} + 2}$, $x \in R$ then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots\ldots + f\left(\frac{2022}{2023}\right)$ equal to [24-Jan-2023 Shift 2]

Options:

A. 2011

B. 1010

C. 2010

D. 1011

Answer: D

Solution:

Solution:

$f(x) = \frac{4^x}{4^x + 2}$

$f(x) + f(1-x) = \frac{4^x}{4^x + 2} + \frac{4^{1-x}}{4^{1-x} + 2}$

$= \frac{4^x}{4^x + 2} + \frac{4}{4 + 2(4^x)}$

$= \frac{4^x}{4^x + 2} + \frac{2}{2 + 4^x}$

$= 1$

$\Rightarrow f(x) + f(1-x) = 1$

Now $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + \ldots\ldots. +$

$\ldots\ldots\ldots. + f\left(1 - \frac{3}{2023}\right) + f\left(1 - \frac{2}{2023}\right) + f\left(1 - \frac{1}{2023}\right)$

Now sum of terms equidistant from beginning and end is 1

Sum $= 1 + 1 + 1 + \ldots\ldots\ldots + 1$ (1011 times)

$= 1011$

Question46

For some a, b, c ∈ ℕ, let $f(x) = ax - 3$ and $g(x) = x^b + c$, $x \in \mathbb{R}$. If $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$ then $(fog)(ac) + (gof)(b)$ is equal to ________.
[25-Jan-2023 Shift 1]

Answer: 2039

Solution:

Solution:
Let fog (x) = h(x)

$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow h(x) = fog(x) = 2x^3 + 7$

$fog(x) = a(x^b + c) - 3$

$\Rightarrow a = 2, b = 3, c = 5$

$\Rightarrow fog(ac) = fog(10) = 2007$

$g(f(x) = (2x - 3)^3 + 5.$

$\Rightarrow gof(b) = gof(3) = 32$

$\Rightarrow$ sum $= 2039$

Question47

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0, 2]. Then the value of m is ______
[25-Jan-2023 Shift 2]

Options:

A. 5

B. 3

C. 2

D. 4

Answer: A

Solution:

Solution:
Since,
$-\sqrt{2} \le \sin x - \cos x \le \sqrt{2}$

$\therefore \ -2 \le \sqrt{2}(\sin x - \cos x) \le 2$
(Assume $\sqrt{2}(\sin x - \cos x) = k$)
$-2 \le k \le 2 \ \ldots$ (i)
$f(x) = \log_{\sqrt{m}}(k + k - 2)$
Given,
$0 \le f(x) \le 2$
$0 \le \log_{\sqrt{min}}(k + m - 2) \le 2$
$1 \le k + m - 2 \le m$
$-m + 3 \le k \le 2 \ldots$ ..ii)
From eq. (i) & (ii), we get $-m + 3 = -2$
$\Rightarrow m = 5$

Question48

The number of functions $f : \{1, 2, 3, 4\} \to \{a \in \mathbb{Z} : |a| \le 8\}$ satisfying $f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1, 2, 3\}$ is
[25-Jan-2023 Shift 2]

Options:

A. 3

B. 4

C. 1

D. 2

Answer: D

Solution:

Solution:
$f : \{1, 2, 3, 4\} \to \{a \in \mathbb{Z} : |a| \le 8\}$
$f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1, 2, 3\}$
$f(n+1)$ must be divisible by n
$f(4) \Rightarrow -6, -3, 0, 3, 6$
$f(3) \Rightarrow -8, -6, -4, -2, 0, 2, 4, 6, 8$
$f(2) \Rightarrow -8, \ldots\ldots\ldots\ldots\ldots, 8$
$f(1) \Rightarrow -8, \ldots\ldots\ldots\ldots\ldots, 8$
$\frac{f(4)}{3}$ must be odd since $f(3)$ should be even therefore 2 solution possible.

Question49

Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is
[25-Jan-2023 Shift 2]

Options:

A. 61

B. 60

C. 58

D. 59

Answer: B

Solution:

Solution:
$f(x) = 2x^n + \lambda$
$f(4) = 133$
$f(5) = 255$
$133 = 2 \times 4^n + \lambda \ldots (1)$
$255 = 2 \times 5^n + \lambda \ldots (2)$
$(2) \quad - (1)$
$122 = 2(5^n - 4^n)$
$\Rightarrow 5^n - 4^n = 61$
$\therefore n = 3 \& \lambda = 5$
Now, $f(3) - f(2) = 2(3^3 - 2^3) = 38$
Number of Divisors is 1, 2, 19, 38; & their sum is 60

Question50

Let $f : R \rightarrow R$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then [29-Jan-2023 Shift 1]

Options:

A. $f(x)$ is many-one in $(-\infty, -1)$

B. $f(x)$ is many-one in $(1, \infty)$

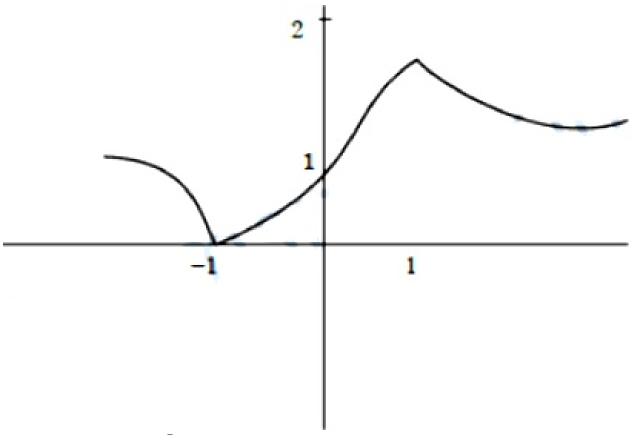
C. $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

D. $f(x)$ is one-one in $(-\infty, \infty)$

Answer: C

Solution:

Solution:



$$f(x)=\frac{(x+1)^2}{x^2+1}=1+\frac{2x}{x^2+1}$$

$$f(x)=1+\frac{2}{x+\frac{1}{x}}$$

Question51

The domain of $f(x)=\frac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}$, $x \in R$ is [29-Jan-2023 Shift 1]

Options:

A. $\mathbb{R}-\{1-3\}$

B. $(2, \infty)-\{3\}$

C. $(-1, \infty)-\{3\}$

D. $\mathbb{R}-\{3\}$

Answer: B

Solution:

Solution:
$x-2>0 \Rightarrow x>2$
$x+1>0 \Rightarrow x>-1$
$x+1 \neq 1 \Rightarrow x \neq 0$ and $x>0$
Denominator
$x^2-2x-3 \neq 0$
$(x-3)(x+1) \neq 0$
$x \neq -1, 3$
So Ans $(2, \infty)-\{3\}$

Question52

Consider a function $f: \mathbb{N} \rightarrow \mathbb{R}$, satisfying $f(1)+2f(2)+3f(3)+\ldots+xf(x)=x(x+1)f(x)$; $x \geq 2$ with $f(1)=1$. Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to

[29-Jan-2023 Shift 2]

Options:

A. 8200

B. 8000

C. 8400

D. 8100

Answer: D

Solution:

Solution:
Given for $x \geq 2$
$f(1) + 2f(2) + \ldots + xf(x) = x(x+1)f(x)$
replace x by $x+1$
$\Rightarrow x(x+1)f(x) + (x+1)f(x+1)$
$= (x+1)(x+2)f(x+1)$
$\Rightarrow \dfrac{x}{f(x+1)} + \dfrac{1}{f(x)} = \dfrac{(x+2)}{f(x)}$
$\Rightarrow xf(x) = (x+1)f(x+1) = \dfrac{1}{2}, x \geq 2$
$f(2) = \dfrac{1}{4}, f(3) = \dfrac{1}{6}$

Now $f(2022) = \dfrac{1}{4044}$
$f(2028) = \dfrac{1}{4056}$
So, $\dfrac{1}{f(2022)} + \dfrac{1}{f(2028)} = 4044 + 4056 = 8100$

Question53

Suppose $f : R \to (0, \infty)$ be a differentiable function such that $5f(x+y) = f(x) \cdot f(y), \forall x, y \in R$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to: [30-Jan-2023 Shift 1]

Options:

A. 6875

B. 6575

C. 6825

D. 6528

Answer: C

Solution:

Solution:
Option (3)
$5f(x+y) = f(x) \cdot f(y)$

$5f(0) = f(0)^2 \Rightarrow f(0) = 5$
$5f(x+1) = f(x) \cdot f(1)$

$\Rightarrow \dfrac{f(x+1)}{f(x)} = \dfrac{f(1)}{5}$

$\Rightarrow \dfrac{f(1)}{f(0)} \cdot \dfrac{f(2)}{f(1)} \cdot \dfrac{f(3)}{f(2)} = \left(\dfrac{f(1)}{5}\right)^3$

$\Rightarrow \dfrac{320}{5} = \dfrac{(f(1))^3}{5^3} \Rightarrow f(1) = 20$

$\therefore 5f(x+1) = 20 \cdot f(x) \Rightarrow f(x+1) = 4f(x)$

$\sum_{n=0}^{5} f(n) = 5 + 5.4 + 5.4^2 + 5.4^3 + 5.4^4 + 5.4^5$

$= \dfrac{5[4^6 - 1]}{3} = 6825$

Question54

Let S = {1, 2, 3, 4, 5, 6}. Then the number of one functions f : S → P(S), where P(S) denote the power set of S, such that f (n) ⊂ f (m) where n < m is ________.
[30-Jan-2023 Shift 1]

Answer: 3240

Solution:

Solution:
Let S = {1, 2, 3, 4, 5, 6}, then the number of one-one functions, $f : S \cdot P(S)$, where $P(S)$ denotes the power set of S, such that $f(n) < f(m)$ where $n < m$ is
$n(S) = 6$

$P(S) = \left\{ \begin{matrix} \varphi & \{1\} & \ldots\{6\} & \{1,2\} & \ldots \\ \{5,6\} & \ldots & \{1,2,3,4,5,6\} & & \end{matrix} \right\}$

-64 elements
case -1
f (6) = S i.e. 1 option,
f (5) = any 5 element subset A of S i.e. 6 options,
f (4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 3 element subset C of B i.e. 4 options,
f(2) = any 2 element subset D of C i.e. 3 options,
f (1) = any 1 element subset E of D or empty subset i.e. 3 options,
Total functions $= 1080$
Case - 2
f (6) = any 5 element subset A of S i.e. 6 options,
f (5) = any 4 element subset B of A i.e. 5 options,
f' (4) = any 3 element subset C of B i.e. 4 options,
f (3) = any 2 element subset D of C i.e. 3 options,
f' (2) = any 1 element subset E of D i.e. 2 options,
f (1) = empty subset i.e. 1 option
Total functions $= 720$
Case -3
f(6) = S
f (5) = any 4 element subset A of' S i.e. 15 options,
f (4) = any 3 element subset B of A i.e. 4 options,
f (3) = any 2 element subset C of B i.e. 3 options,
f (2) = any 1 element subset D of C i.e. 2 options,
f (1) = empty subset i.e. 1 option

Total functions $= 360$
Case -4

$f(6) = S$
$f(5) =$ any 5 element subset A of S i.e. 6 options,
$f(4) =$ any 3 element subset B of A i.e. 10 options,
$f(3) =$ any 2 element subset C of B i.e. 3 options,
$f(2) =$ any 1 element subset D of C i.e. 2 options,
$f(1) =$ empty subset i.e. 1 option
Total functions $= 360$
Case -5
$f(6) = S$
$f(5) =$ any 5 element subset A of S i.e. 6 options,
$f(4) =$ any 4 element subset B of A i.e. 5 options,
$f(3) =$ any 2 element subset C of B i.e. 6 options,
$f(2) =$ any 1 element subset D of C i.e. 2 options,
$f(1) =$ empty subset i.e. 1 option
Total functions $= 360$
Case - 6
$f(6) = S$
$f(5) =$ any 5 element subset A of S i.e. 6 options,
$f(4) =$ any 4 element subset B of A i.e. 5 options,
$f(3) =$ any 3 element subset C of B i.e. 4 options,
$f(2) =$ any 1 element subset D of C i.e. 3 options,
$f(1) =$ empty subset i.e. 1 option
Total functions $= 360$
$\therefore$ Number of such functions $= 3240$

Question55

Let $f^1(x) = \frac{3x+2}{2x+3}, x \in R - \left\{\frac{-3}{2}\right\}$
For $n \geq 2$, define $f^n(x) = f^1 0 f^{n-1}(x)$.
If $f^5(x) = \frac{ax+b}{bx+a}$, gcd(a, b) = 1, then a + b is equal to ________.
[30-Jan-2023 Shift 1]

Answer: 3125

Solution:

Solution:

$f^1(x) = \frac{3x+2}{2x+3}$

$\Rightarrow f^2(x) = \frac{13x+12}{12x+13}$

$\Rightarrow f^3(x) = \frac{63x+62}{62x+63}$

$\therefore f^5(x) = \frac{1563x+1562}{1562x+1563}$

$a + b = 3125$

Question56

The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is
[30-Jan-2023 Shift 2]

Options:

A. $[\sqrt{5}, \sqrt{10}]$

B. $[2\sqrt{2}, \sqrt{11}]$

C. $[\sqrt{5}, \sqrt{13}]$

D. $[\sqrt{2}, \sqrt{7}]$

Answer: A

Solution:

Solution:

$y^2 = 3 - x + 2 + x + 2\sqrt{(3-x)(2+x)}$

$= 5 + 2\sqrt{6 + x - x^2}$

$y^2 = 5 + 2\sqrt{\frac{25}{4} - \left(x - \frac{1}{2}\right)^2}$

$y_{max} = \sqrt{5+5} = \sqrt{10}$

$y_{min} = \sqrt{5}$

Question57

Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to ______.
[30-Jan-2023 Shift 2]

Answer: 432

Solution:

Solution:

$f(1) = 1;\ f(9) = f(3) \times f(3)$

i.e., $f(3) = 1$ or 3

Total function $= 1 \times 6 \times 2 \times 6 \times 6 \times 1 = 432$

Question58

If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\le x$, is $[2, 6)$, then its range is
[31-Jan-2023 Shift 1]

Options:

A. $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

B. $\left(\frac{5}{26}, \frac{2}{5}\right]$

C. $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

D. $\left(\frac{5}{37}, \frac{2}{5}\right]$

Answer: D

Solution:

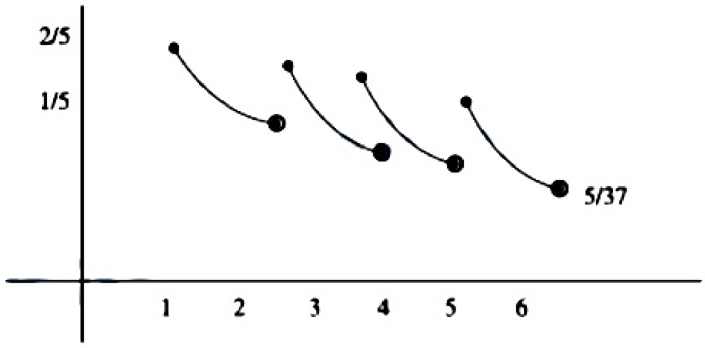
Solution:

$f(x) = \frac{2}{1+x^2}\ x \in [2, 3)$

$f(x) = \frac{3}{1+x^2}\ x \in [3, 4)$

$f(x) = \frac{4}{1+x^2}\ x \in [4, 5)$

$f(x) = \frac{5}{1+x^2}\ x \in [5, 6)$



$\left(\frac{5}{37}, \frac{2}{5}\right]$

Question59

The absolute minimum value, of the function $f(x) = x^2 - x + 1 \mid + [x^2 - x + 1]$, where [t] denotes the greatest integer function, in the interval [−1, 2], is :
[31-Jan-2023 Shift 2]

Options:

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. $\frac{1}{4}$

D. $\frac{5}{4}$

Answer: A

Solution:

Solution:

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]; x \in [-1, 2]$

Let $g(x) = x^2 - x + 1$

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\because |x^2 - x + 1|$ and $[x^2 - x + 2]$

Both have minimum value at $x = 1/2$

$\Rightarrow$ Minimum $f(x) = \frac{3}{4} + 0$

$= \frac{3}{4}$

Question60

Let $f : \mathbb{R} - \{2, 6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$. Then range of f is
[31-Jan-2023 Shift 2]

Options:

A. $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

B. $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

C. $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

D. $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

Answer: A

Solution:

Solution:

Let $y = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

By cross multiplying

$yx^2 - 8xy + 12y - x^2 - 2x - 1 = 0$

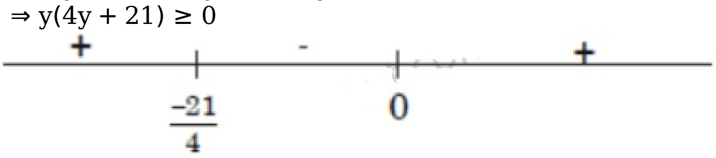
$x^2(y - 1) - x(8y + 2) + (12y - 1) = 0$

Case 1, $y \neq 1$

$D \geq 0$

$\Rightarrow (8y + 2)^2 - 4(y - 1)(12y - 1) \geq 0$

$\Rightarrow y(4y + 21) \geq 0$



$y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty) - \{1\}$

Case 2, $y = 1$

$x^2 + 2x + 1 = x^2 - 8x + 12$

$10x = 11$

$x = \frac{11}{10}$ So, y can be 1

Hence $y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$

Question61

Let $f : R - \{0, 1\} \rightarrow R$ be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then $f(2)$ is equal to :
[1-Feb-2023 Shift 2]

Options:

A. $\frac{9}{2}$

B. $\frac{9}{4}$

C. $\frac{7}{4}$

D. $\frac{7}{3}$

Answer: B

Solution:

Solution:

$f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$

$x = 2 \Rightarrow f(2) + f(-1) = 3$

$x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0 \ldots (2)$

$x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2} \ldots (3)$

$(1) + (3) - (2) \Rightarrow 2f(2) = \frac{9}{2}$

$\therefore f(2) = \frac{9}{4}$

Question62

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x - 1$ and $g : \mathbb{R} - \{1, -1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$. Then the function fog is :
[26-Jun-2022-Shift-2]

Options:

A. one-one but not onto

B. onto but not one-one

C. both one-one and onto

D. neither one-one nor onto

Answer: D

Solution:

$f: R \to R$ defined as

$f(x) = x - 1$ and $g: R \to \{1, -1\} \to R,\ g(x) = \dfrac{x^2}{x^2 - 1}$

Now $fog(x) = \dfrac{x^2}{x^2-1} - 1 = \dfrac{1}{x^2-1}$

$\therefore$ Domain of $fog(x) = R - \{-1, 1\}$

And range of $fog(x) = (-\infty, -1] \cup (0, \infty)$

Now, $\dfrac{d}{dx}(\text{fog}(x)) = \dfrac{-1}{x^2-1} \cdot 2x = \dfrac{2x}{1-x^2}$

$\therefore \dfrac{d}{dx}(f \circ g(x)) > 0$ for $\dfrac{2x}{(1-x)(1+x)} > 0$

$\Rightarrow \dfrac{x}{(x-1)(x+1)} < 0$

$\therefore x \in (-\infty, -1) \cup (0, 1)$

and $\dfrac{d}{dx}(f \circ g(x)) < 0$ for $x \in (-1, 0) \cup (1, \infty)$

$\therefore fog(x)$ is neither one-one nor onto.

Question63

Let $f : R \to R$ be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x} + e}$.
Then
$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \ldots + f\left(\frac{99}{100}\right)$$
is equal to____
[27-Jun-2022-Shift-1]

Answer: 99

Solution:

Solution:
Given,

$f(x) = \frac{2e^{2x}}{e^{2x} + e}$

$\therefore f(1 - x) = \frac{2e^{2(1-x)}}{e^{2(1-s)} + e}$

$= \frac{2 \cdot \frac{e^2}{e^{2x}}}{\frac{e^2}{e^{2\pi}} + e}$

$= \frac{2e^2}{e^2 + e^{2x} \cdot e}$

$= \frac{2e^2}{e(e + e^{2x})}$

$= \frac{2e}{e + e^{2\Delta}}$

$\therefore f(x) + f(1 - x) = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e}{e^{2x} + e}$

$= \frac{2(e^{2x} + e)}{e^{2x} + e}$

$= 2 \ldots\ldots (1)$

Now,

$f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right)$

$= f\left(\frac{1}{100}\right) + f\left(1 - \frac{1}{100}\right)$

$= 2$ [as $f(x) + f(1 - x) = 2$]

$f\left(\frac{2}{100}\right) + f\left(1 - \frac{2}{100}\right) = 2$

⁞

$f\left(\frac{49}{100}\right) + f\left(1 - \frac{49}{100}\right) = 2$

∴ Total sum $= 49 \times 2$

Remaining term $= f\left(\frac{50}{100}\right) = f\left(\frac{1}{2}\right)$

Put $x = \frac{1}{2}$ in equation (1), we get

$f\left(\frac{1}{2}\right) + f\left(1 - \frac{1}{2}\right) = 2$

$\Rightarrow 2f\left(\frac{1}{2}\right) = 2$

$\Rightarrow f\left(\frac{1}{2}\right) = 1$

$\therefore$ Sum $= 49 \times 2 + 1 = 99$

Question64

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define $f : S \rightarrow S$ as

$$f(n) = \begin{cases} 2n & , \text{if } n = 1, 2, 3, 4, 5 \\ 2n - 11 & , \text{if } n = 6, 7, 8, 9, 10, \end{cases}$$

Let $g : S \rightarrow S$ be a function such that $fog(n) = \begin{cases} n+1 & , \text{if } n \text{ is odd} \\ n-1 & , \text{if } n \text{ is even} \end{cases}$

Then $g(10)g(1) + g(2) + g(3) + g(4) + g(5))$ is equal to
[27-Jun-2022-Shift-2]

Answer: 190

Solution:

Solution:

$$\because f(n) = \begin{cases} 2n & n = 1\ 2\ 3\ 4\ 5 \\ 2n - 11 & n = 6\ 7\ 8\ 9\ 10 \end{cases}.$$

$\therefore f(1) = 2, f(2) = 4, \ldots\ldots, f(5) = 10$
and $f(6) = 1, f(7) = 3, f(8) = 5, \ldots\ldots., f(10) = 9$

Now, $f(g(n)) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}.$

$f(g(10)) = 9 \Rightarrow g(10) = 10$
$f(g(1)) = 2 \Rightarrow g(1) = 1$
. $f(g(2)) = 1 \Rightarrow g(2) = 6$
$\therefore f(g(3)) = 4 \Rightarrow g(3) = 2$
$f(g(4)) = 3 \Rightarrow g(4) = 7$
$f(g(5)) = 6 \Rightarrow g(5) = 3$
$\therefore g(10)g(1) + g(2) + g(3) + g(4) + g(5)) = 190$

Question65

Let a function $f : N \rightarrow N$ be defined by

$$f(n) = \begin{bmatrix} 2n & n = 2, 4, 6, 8, \ldots\ldots \\ n-1 & n = 3, 7, 11, 15, \ldots\ldots \\ \frac{n+1}{2} & n = 1, 5, 9, 13, \ldots\ldots. \end{bmatrix}$$

then, f is
[28-Jun-2022-Shift-1]

Options:

A. one-one but not onto

B. onto but not one-one

C. neither one-one nor onto

D. one-one and onto

Answer: D

Solution:

Solution:

When $n = 1, 5, 9, 13$ then $\frac{n+1}{2}$ will give all odd numbers.
When $n = 3, 7, 11, 15......$
$n - 1$ will be even but not divisible by 4
When $n = 2, 4, 6, 8...$
Then $2n$ will give all multiples of 4
So range will be N.
And no two values of n give same y, so function is one-one and onto.

Question66

The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f(a) + 2f(b) - f(c) = f(d)$ is : [28-Jun-2022-Shift-2]

Options:

A. $\frac{1}{24}$

B. $\frac{1}{40}$

C. $\frac{1}{30}$

D. $\frac{1}{20}$

Answer: D

Solution:

Solution:

Number of one-one function from $\{a, b, c, d\}$ to set $\{1, 2, 3, 4, 5\}$ is ${}^5P_4 = 120 n(s)$.
The required possible set of value $(f(a), f(b), f(c), f(d))$ such that $f(a) + 2f(b) - f(c) = f(d)$ are $(5, 3, 2, 1)$, $(5, 1, 2, 3)$, $(4, 1, 3, 5)$, $(3, 1, 4, 5)$, $(5, 4, 3, 2)$ and $(3, 4, 5, 2)$
$\therefore n(E) = 6$
$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{6}{120} = \frac{1}{20}$

Question67

Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \to S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \ \forall (a, b) \in S \times S\}$ is____ [28-Jun-2022-Shift-2]

Answer: 37

Solution:

Solution:
There are 16 ordered pairs in $S \times S$. We write all these ordered pairs in 4 sets as follows.
$A = \{(1, 1)\}$
$B = \{(1, 4), (2, 4), (3, 4)(4, 4), (4, 3), (4, 2), (4, 1)\}$
$C = \{(1, 3), (2, 3), (3, 3), (3, 2), (3, 1)\}$
$D = \{(1, 2), (2, 2), (2, 1)\}$
All elements of set B have image 4 and only element of A has image 1 .
All elements of set C have image 3 or 4 and all elements of set D have image 2 or 3 or 4 .
We will solve this question in two cases.
Case I: When no element of set C has image 3 .
Number of onto functions $= 2$ (when elements of set D have images 2 or 3)
Case II: When atleast one element of set C has image 3 .
Number of onto functions $= (2^3 - 1)(1 + 2 + 2) = 35$
Total number of functions $= 37$

Question68

The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is : [29-Jun-2022-Shift-1]

Options:

A. $R - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

B. $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

C. $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

D. $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Answer: D

Solution:

Solution:

$$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$$

$$\Rightarrow -\frac{\pi}{2} \le \sin^{-1}\left(\frac{1}{4x^2-1}\right) \le \frac{\pi}{2}$$

$$\Rightarrow -1 \le \frac{1}{4x^2-1} \le 1$$

$$\therefore \frac{1}{4x^2-1} + 1 \ge 0$$

$$\Rightarrow \frac{1 + 4x^2 - 1}{4x^2 - 1} \ge 0$$

$\Rightarrow \frac{4x^2}{4x^2-1} \geq 0$

$\Rightarrow \frac{4x^2}{(2x+1)(2x-1)} \geq 0 \ldots\ldots (1)$

$\therefore x \in \left(-\alpha, -\frac{1}{2}\right) \cup \{0\} \cup \left(\frac{1}{2}, \alpha\right)$

And $\frac{1}{4x^2-1} - 1 \leq 0$

$\Rightarrow \frac{1-4x^2+1}{4x^2-1} \leq 0$

$\Rightarrow \frac{2-4x^2}{4x^2-1} \leq 0$

$\Rightarrow \frac{2x^2-1}{4x^2-1} \geq 0$

$\Rightarrow \frac{(\sqrt{2}x+1)(\sqrt{2}x-1)}{(2x+1)(2x-1)} \geq 0$

$x \in \left(-\alpha, -\frac{1}{\sqrt{2}}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{\sqrt{2}}, \alpha\right)$

From (3) and (4), we get

$\therefore x \in \left[-\alpha, -\frac{1}{\sqrt{2}}\right) \cup \left[\frac{1}{\sqrt{2}}, \alpha\right) \cup \{0\}$

Question69

Let c, k ∈ R. If $f(x) = (c+1)x^2 + (1-c^2)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all x, y ∈ R, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to____ [29-Jun-2022-Shift-1]

Answer: 3395

Solution:

Solution:

f (x) is polynomial

Put y = 1 / x in given functional equation we get

$f\left(x + \frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) - 1$

$\Rightarrow (c+1)\left(x + \frac{1}{x}\right)^2 + (1-c^2)\left(x + \frac{1}{x}\right) + 2K$

$= (c+1)x^2 + (1-c^2)x + 2K + (c+1)\frac{1}{x^2} + (1-c^2)\frac{1}{x} + 2K - 1$

$\Rightarrow 2(c+1) = 2K - 1 \ldots\ldots (1)$

and put x = y = 0 we get

$f(0) = 2 + f(0) - 0 \Rightarrow f(0) = 0 \Rightarrow k = 0$

$\therefore k = 0$ and $2c = -3 \Rightarrow c = -3/2$

$f(x) = -\frac{x^2}{2} - \frac{5x}{4} = \frac{1}{4}(5x + 2x^2)$

$\left|2\sum_{i=1}^{20} f(i)\right| = \left|\frac{-2}{4}\left(\frac{5.20.21}{2} + \frac{2.20.21.41}{6}\right)\right|$

$= \left|\frac{-1}{2}(2730 + 5740)\right|$

$= \left|-\frac{6790}{2}\right| = 3395.$

Question70

Let $f(x)$ and $g(x)$ be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$ and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is_____
[29-Jun-2022-Shift-2]

Answer: 18

Solution:

Solution:
$f(g(x) = 8x^2 - 2x.$
$g(f(x) = 4x^2 + 6x + 1.$
$\text{So, } g(x) = 2x - 1$
$\& f(x) = 2x^2 + 3x + 1$
$f(2) = 8 + 6 + 1 = 15$
Ans. 18

Question71

The domain of the function
$$f(x) = \frac{\cos^{-1}\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\log_e(x^2 - 3x + 2)} \text{ is :}$$
[24-Jun-2022-Shift-1]

Options:

A. $(-\infty, 1) \cup (2, \infty)$

B. $(2, \infty)$

C. $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

D. $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Answer: D

Solution:

Solution:

$-1 \leq \frac{x^2-5x+6}{x^2-9} \leq 1$ and $x^2-3x+2>0, \neq 1$

$\frac{(x-3)(2x+1)}{x^2-9} \geq 0 \mid \frac{5(x-3)}{x^2-9} \geq 0$

The solution to this inequality is

$x \in \left[\frac{-1}{2}, \infty\right) - \{3\}$

for $x^2-3x+2>0$ and $\neq 1$

$x \in (-\infty, 1) \cup (2, \infty) - \left\{\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right\}$

Combining the two solution sets (taking intersection)

$x \in \left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right\}$

Question72

The number of one-one functions f : {a, b, c, d} → {0, 1, 2, ..., 10} such that 2f (a) − f (b) + 3f (c) + f (d) = 0 is
[24-Jun-2022-Shift-1]

Answer: 31

Solution:

$-1 \leq \frac{x^2-5x+6}{x^2-9} \leq 1$ and $x^2-3x+2>0, \neq 1$

$\frac{(x-3)(2x+1)}{x^2-9} \geq 0 \mid \frac{5(x-3)}{x^2-9} \geq 0$

The solution to this inequality is

$x \in \left[\frac{-1}{2}, \infty\right) - \{3\}$

for $x^2-3x+2>0$ and $\neq 1$

$x \in (-\infty, 1) \cup (2, \infty) - \left\{\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right\}$

Combining the two solution sets (taking intersection)

$x \in \left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right\}$

f(d) can't be 9 and 10 as if $f(d)=9$ or 10 then $f(b)=2+9=11$ or $f(b)=2+10=12$, which is not possible as here any function's maximum value can be 10 .

$\therefore$ Total possible functions when $f(c)=0$ and $f(a)=1$ are $=7$

(2) When $f(c)=0$ and $f(a)=2$ then

$3\times 0+2\times 2+f(d)=f(b)$

$\Rightarrow 4+f(d)=f(b)$

$\therefore$ possible value of $f(d)=1,3,4,5,6$

$\therefore$ Total possible functions in this case $=5$

(3) When $f(c)=0$ and $f(a)=3$ then

$3\times 0+2\times 3+f(d)=f(b)$

$\Rightarrow 6+f(d)=f(b)$

$\therefore$ Possible value of $f(d)=1,2,4$

$\therefore$ Total possible functions in this case $=3$

(4) When $f(c)=0$ and $f(a)=4$ then

$3\times 0+2\times 4+f(d)=f(b)$

$\Rightarrow 8+f(d)=f(b)$

$\therefore$ Possible value of $f(d)=1,2$

$\therefore$ Total possible functions in this case $=2$

(5) When $f(c)=0$ and $f(a)=5$ then

$3\times 0+2\times 5+f(d)=f(b)$

$\Rightarrow 10+f(d)=f(b)$

Possible value of $f(d)$ can be 0 but $f(c)$ is already zero. So, no value to $f(d)$ can satisfy.

$\therefore$ No function is possible in this case.

$\therefore$ Total possible functions when f(c)=0 and f(a)=1,2,3 and 4 are =7+5+3+2=17

Case II:

(1) When f(c)=1 and f(a)=0 then

3×1+2×0+f(d)=f(b)

⇒3+f(d)=f(b)

$\therefore$ Possible value of f(d)=2,3,4,5,6,7

$\therefore$ Total possible functions in this case =6

(2) When f(c)=1 and f(a)=2 then

3×1+2×2+f(d)=f(b)

$\Rightarrow 7+f(d)=f(b)$

$\therefore$ Possible value of f(d)=0,3

$\therefore$ Total possible functions in this case =2

(3) When f(c)=1 and f(a)=3 then

$3\times1+2\times3+f(d)=f(b)$

$\Rightarrow 9+f(d)=f(b)$

$\therefore$ Possible value of f(d)=0

$\therefore$ Total possible functions in this case =1

$\therefore$ Total possible functions when f(c)=1 and f(a)=0,2 and 3 are =6+2+1=9

Case III:

(1) When f(c)=2 and f(a)=0 then

$3\times2+2\times0+f(d)=f(b)$

$\Rightarrow 6+f(d)=f(b)$

$\therefore$ Possible values of f(d)=1,3,4

$\therefore$ Total possible functions in this case =3

(2) When f(c)=2 and f(a)=1 then,

$3\times2+2\times1+f(d)=f(b)$

$\Rightarrow 8+f(d)=f(b)$

$\therefore$ Possible values of f(d)=0

$\therefore$ Total possible function in this case =1

$\therefore$ Total possible functions when f(c)=2 and f(a)=0,1 are =3+1=4

Case IV:

(1) When f(c)=3 and f(a)=0 then

$3\times3+2\times0+f(d)=f(b)$

$\Rightarrow 9+f(d)=f(b)$

$\therefore$ Possible values of f(d)=1

$\therefore$ Total one-one functions from four cases

=17+9+4+1=31

Question73

Let R_1 and R_2 be relations on the set {1, 2, ..., 50} such that
$R_1 = \{ (p, p^n) : p.$ is a prime and $n \ge 0$ is an integer $\}$ and
$R_2 = \{ (p, p^n) : p.$ is a prime and $n = 0$ or $1\}$
Then, the number of elements in $R_1 - R_2$ is___
[28-Jun-2022-Shift-1]

Answer: 8

Solution:

$R_1 - R_2 = \{(2, 2^2), (2, 2^3), (2, 2^4), (2, 2^5), (3, 3^2), (3, 3^3), (5, 5^2), (7, 7^2)\}$
So number of elements $= 8$

Question74

Let $R_1 = \{(a, b) \in N \times N : |a - b| \le 13\}$ and $R_2 = \{(a, b) \in N \times N : |a - b| \ne 13\}$. Then on N :
[28-Jun-2022-Shift-2]

Options:

A. Both R_1 and R_2 are equivalence relations

B. Neither R_1 nor R_2 is an equivalence relation

C. R_1 is an equivalence relation but R_2 is not

D. R_2 is an equivalence relation but R_1 is not

Answer: B

Solution:

Solution:
$R_1 = \{(a, b) \in N \times N : |a - b| \le 13\}$ and
$R_2 = \{(a, b) \in N \times N : |a - b| \ne 13\}$
In $R_1 : \because |2 - 11| = 9 \le 13$
$\therefore$ $(2, 11) \in R_1$ and $(11, 19) \in R_1$ but $(2, 19) \notin R_1$
$\therefore$ R_1 is not transitive
Hence R_1 is not equivalence
In R_2 : $(13, 3) \in R_2$ and $(3, 26) \in R_2$ but $(13, 26) \notin R_2 (\because |13 - 26| = 13)$
$\therefore R_2$ is not transitive
Hence R_2 is not equivalence.

Question75

The probability that a relation R from {x, y} to {x, y} is both symmetric and transitive, is equal to
[29-Jun-2022-Shift-2]

Options:

A. $\frac{5}{16}$

B. $\frac{9}{16}$

C. $\frac{11}{16}$

D. $\frac{13}{16}$

Answer: A

Solution:

Solution:
Total no. of relations $= 2^{2 \times 2} = 16$
Fav. relation $= \varphi, \{(x, x)\}, \{(y, y)\}, \{(x, x)(y, y)\}$
$\{(x, x), (y, y), (x, y)(y, x)\}$
$\text{Prob.} = \frac{5}{16}$

Question76

The number of bijective functions $f : \{1, 3, 5, 7, ..., 99\} \rightarrow \{2, 4, 6, 8, ... 100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq .... f(99)$, is
[25-Jul-2022-Shift-2]

Options:

A. ${}^{50}P_{17}$

B. ${}^{50}P_{33}$

C. $33! \times 17!$

D. $\frac{50!}{2}$

Answer: B

Solution:

Solution:
As function is one-one and onto, out of 50 elements of domain set 17 elements are following restriction
$f(3) > f(9) > f(15) ...... > f(99)$
So number of ways $= {}^{50}C_{17} \cdot 1.33\,!$
$= {}^{50}P_{33}$

Question77

Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p$, $p \neq 0$, and $f(1) = \frac{1}{3}$. If the equations $f(x) = 0$ and $f \circ f \circ f \circ f(x) = 0$ have a common real root, then $f(-3)$ is equal to___
[25-Jul-2022-Shift-2]

Answer: 25

Solution:

Solution:
Let $f(x) = (x - \alpha)(x - \beta)$
It is given that $f(0) = p \Rightarrow \alpha\beta = p$
and $f(1) = \frac{1}{3} \Rightarrow (1 - \alpha)(1 - \beta) = \frac{1}{3}$
Now, let us assume that, α is the common root of $f(x) = 0$ and $fofofof(x) = 0$
$fofofof(x) = 0$
$\Rightarrow fofof(0) = 0$
$\Rightarrow fof(p) = 0$
So, $f(p)$ is either α or β.
$(p - \alpha)(p - \beta) = \alpha$
$(\alpha\beta - \alpha)(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)(\alpha - 1)\beta = 1(\because \alpha \neq 0)$
So, $\beta = 3$
$(1 - \alpha)(1 - 3) = \frac{1}{3}$
$\alpha = \frac{7}{6}$
$f(x) = \left(x - \frac{7}{6}\right)(x - 3)$
$f(-3) = \left(-3 - \frac{7}{6}\right)(3 - 3) = 25$

Question78

Let $f : R \rightarrow R$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to:
[26-Jul-2022-Shift-1]

Options:

A. 4

B. 10

C. 11

D. 16

Answer: B

Solution:

Solution:
$f(3x) - f(x) = x \ldots\ldots (1)$
$x \rightarrow \frac{x}{3}$
$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3} \ldots\ldots (2)$
Again $x \rightarrow \frac{x}{3}$
$f\left(\frac{x}{3}\right) - f\left(\frac{x}{9}\right) = \frac{x}{3^2} \ldots\ldots$
Similarly

$f\left(\frac{x}{3^{n-2}}\right) - f\left(\frac{x}{3^{n-1}}\right) = \frac{x}{3^{n-1}} \dots (n)$

Adding all these and applying $n \to \infty$

$\lim_{n \to \infty}\left(f(3x) - f\left(\frac{x}{3^{n-1}}\right)\right) = x\left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots\right)$

$f(3x) - f(0) = \frac{3x}{2}$

Putting $x = \frac{8}{3}$

$f(8) - f(0) = 4$

$\Rightarrow f(0) = 3$

Putting $x = \frac{14}{3}$

$f(14) - 3 = 7 \Rightarrow f(14) = 0$

Question79

The domain of the function

$f(x) = \sin^{-1}[2x^2 - 3] + \log_2\left(\log_{\frac{1}{2}}(x^2 - 5x + 5)\right)$, where [t] is the greatest integer function, is :

[27-Jul-2022-Shift-2]

Options:

A. $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$

B. $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$

C. $\left(1, \frac{5-\sqrt{5}}{2}\right)$

D. $\left[1, \frac{5+\sqrt{5}}{2}\right)$

Answer: C

Solution:

Solution:

$-1 \le 2x^2 - 3 < 2$

or $2 \le 2x^2 < 5$

or $1 \le x^2 < \frac{5}{2}$

$x \in \left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right)$

$\log_{\frac{1}{2}}(x^2 - 5x + 5) > 0$

$0 < x^2 - 5x + 5 < 1$

$x^2 - 5x + 5 > 0 \& x^2 - 5x + 4 < 0$

$x \in \left(-\infty, \frac{5-\sqrt{5}}{2}\right) \cup \left(\frac{5+\sqrt{5}}{2}, \infty\right)$

$\& x \in (-\infty, 1) \cup (4, \infty)$

Taking intersection

$x \in \left(1, \frac{5-\sqrt{5}}{2}\right)$

Question80

The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \le 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \le (x - 3)^2 + 1$, for every $x \in A$, is ________.
[27-Jul-2022-Shift-2]

Answer: 1440

Solution:

Solution:
$A = \{x \in N, x^2 - 10x + 9 \le 0\}$
$= \{1, 2, 3, \ldots, 9\}$
$B = \{1, 4, 9, 16, \ldots\ldots\}$
$f(x) \le (x - 3)^2 + 1$
$f(1) \le 5, f(2) \le 2, \ldots\ldots\ldots f(9) \le 37$
$x = 1$ has 2 choices
$x = 2$ has 1 choice
$x = 3$ has 1 choice
$x = 4$ has 1 choice
$x = 5$ has 2 choices
$x = 6$ has 3 choices
$x = 7$ has 4 choices
$x = 8$ has 5 choices
$x = 9$ has 6 choices
$\therefore$ Total functions $= 2 \times 1 \times 1 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 1440$

Question81

Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is :
[28-Jul-2022-Shift-1]

Options:

A. $\left(-\infty, \frac{1}{4}\right]$

B. $\left[-\frac{1}{4}, \infty\right)$

C. $(-1/3, \infty)$

D. $\left(-\infty, \frac{1}{3}\right]$

Answer: B

Solution:

Solution:

$-1 \le \frac{x^2 - 4x + 2}{x^2 + 3} \le 1$

$\Rightarrow -x^2 - 3 \le x^2 - 4x + 2 \le x^2 + 3$

$\Rightarrow 2x^2 - 4x + 5 \ge 0 \quad -4x \le 1$

$x \in R \& x \ge -\frac{1}{4}$

So domain is $\left[-\frac{1}{4}, \infty\right)$

Question82

Let α, β and γ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x$, $x \in R$ and $g : R \to R$. be such that $g(f(x)) = x$ for all $x \in R$. If $a_1, a_2, a_3, \ldots, a_n$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$ is equal to:

[28-Jul-2022-Shift-1]

Options:

A. 0

B. 3

C. 9

D. 27

Answer: A

Solution:

Solution:

$f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$

$\frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n} = 0$

∴ First and last term, second and second last and so on are equal in magnitude but opposite in sign.

$f(x) = \alpha x^5 + \beta x^3 + \gamma x$

$\sum_{i=1}^{n} f(a_i) = \alpha(a_1^5 + a_2^5 + a_3^5 + \ldots + a_n^5) + \beta(a_1^3 + a_2^3 + \ldots + a_n^3) + \gamma(a_1 + a_2 + \ldots + a_n)$

$= 0\alpha + 0\beta + 0\gamma$

$= 0$

$\therefore f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right) = \frac{1}{n}\sum_{i=1}^{n} f(a_i) = 0$

Question83

The number of elements in the set

$S = \left\{ x \in \mathbb{R} : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x} \right\}$ is:

[29-Jul-2022-Shift-2]

Options:

A. 1

B. 3

C. 0

D. infinite

Answer: A

Solution:

Solution:

$2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}$

L.H.S ≤ 2. & R.H.S. ≥ 2

Hence L.H.S $= 2$ & R.H.S $= 2$

$2\cos\left(\frac{x^2+x}{6}\right) = 2 \quad 4^x + 4^{-x} = 2$

Check $x = 0$ Possible hence only one solution.

Question84

The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is:

[29-Jul-2022-Shift-2]

Options:

A. $[1, \infty)$

B. $[-1, 2]$

C. $[-1, \infty)$

D. $(-\infty, 2]$

Answer: C

Solution:

Solution:

$f(x) = \sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$

$-1 \le \frac{x^2-3x+2}{x^2+2x+7} \le 1$

$\frac{x^2-3x+2}{x^2+2x+7} \le 1$

$x^2-3x+2 \le x^2+2x+7$

$5x \ge -5$

$x \ge -1$

And $\frac{x^2-3x+2}{x^2+2x+7} \ge -1$

$x^2-3x+2 \ge -x^2-2x-7$

$2x^2-x+9 \ge 0$

$x \in R$
(i) ∩ (ii)
Domain $\in [-1, \infty)$

Question85

The total number of functions, $f : \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to :

[25-Jul-2022-Shift-1]

Options:

A. 60

B. 90

C. 108

D. 126

Answer: B

Solution:

Solution:
Given, $f(1) + f(2) = f(3)$
It means $f(1)$, $f(2)$ and $f(3)$ are dependent on each other. But there is no condition on $f(4)$, so $f(4)$ can be $f(4) = 1, 2, 3, 4, 5, 6.$
For $f(1)$, $f(2)$ and we have to find how many functions possible which will satisfy the condition $f(1) + f(2) = f(3)$
Case 1:
When $f(3) = 2$ then possible values of $f(1)$ and $f(2)$ which satisfy $f(1) + f(2) = f(3)$ is $f(1) = 1$ and $f(2) = 1$.
And $f(4)$ can be $= 1, 2, 3, 4, 5, 6$
$\therefore$ Total possible functions $= 1 \times 6 = 6$
Case 2 :

When $f(3) = 3$ then possible values
(1) $f(1) = 1$ and $f(2) = 2$
(2) $f(1) = 2$ and $f(2) = 1$
And $f(4)$ can be $= 1, 2, 3, 4, 5, 6.$
$\therefore$ Total functions $= 2 \times 6 = 12$
Case 3 :
When $f(3) = 4$ then
(1) $f(1) = 1$ and $f(2) = 3$
(2) $f(1) = 2$ and $f(2) = 2$
(3) $f(1) = 3$ and $f(2) = 1$
And $f(4)$ can be $= 1, 2, 3, 4, 5, 6$
$\therefore$ Total functions $= 3 \times 6 = 18$
Case 4 :
When $f(3) = 5$ then
(1) $f(1) = 1$ and $f(4) = 4$
(2) $f(1) = 2$ and $f(4) = 3$
(3) $f(1) = 3$ and $f(4) = 2$
(4) $f(1) = 4$ and $f(4) = 1$
And $f(4)$ can be $= 1, 2, 3, 4, 5$ and 6
$\therefore$ Total functions $= 4 \times 6 = 24$
Case 5 :
When $f(3) = 6$ then
(1) $f(1) = 1$ and $f(2) = 5$
(2) $f(1) = 2$ and $f(2) = 4$
(3) $f(1) = 3$ and $f(2) = 3$
(4) $f(1) = 4$ and $f(2) = 2$
(5) $f(1) = 5$ and $f(2) = 1$
And $f(4)$ can be $= 1, 2, 3, 4, 5$ and 6

∴ Total possible functions $= 5 \times 6 = 30$
∴ Total functions from those 5 cases we get $= 6 + 12 + 18 + 24 + 30 = 90$

Question86

Let $f : N \to R$ be a function such that $f(x + y) = 2f(x)f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which
$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1)$
holds, is :
[25-Jun-2022-Shift-1]

Options:

A. 2

B. 3

C. 4

D. 6

Answer: C

Solution:

Solution:

Given,

$f(x+y) = 2f(x)f(y)$

and $f(1) = 2$

For $x = 1$ and $y = 1$,

$f(1+1) = 2f(1)f(1)$

$\Rightarrow f(2) = 2(f(1))^2 = 2(2)^2 = 2^3$

For $x = 1, y = 2$

$f(1+2) = 2f(1)y(2)$

$\Rightarrow f(3) = 2 \cdot 2 \cdot 2^3 = 2^5$

For $x = 1, y = 3$

$f(1+3) = 2f(1)f(3)$

$\Rightarrow f(4) = 2 \cdot 2 \cdot 2^5 = 2^7$

For $x = 1, y = 4$

$f(1+4) = 2f(1)f(4)$

$\Rightarrow f(5) = 2.2 \cdot 2^7 = 2^9 \ldots.$

Also given

$\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20} - 1)$

$\Rightarrow f(\alpha+1)+f(\alpha+2)+f(\alpha+3)+\ldots+f(\alpha+10)=\frac{512}{3}(2^{20}-1)$

$\Rightarrow f(\alpha+1)+f(\alpha+2)+f(\alpha+3)+\ldots+f(\alpha+10)=\frac{2^9((2^2)^{10}-1)}{2^2-1}$

This represent a G.P with first term $=2^9$ and common ratio $=2^2$

$\therefore$ First term $=f(\alpha+1)=2^9\ldots(2)$

From equation (1), $f(5)=2^9$

$\therefore$ From (1) and (2), we get

$f(\alpha+1)=2^9=f(5)$

$\Rightarrow f(\alpha+1)=f(5)$

$\Rightarrow f(\alpha+1)=f(4+1)$

Comparing both sides we get, $\alpha=4$

Question87

Let f : R → R be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)(2+x^{25})\right)^{\frac{1}{50}}$. If the function g(x) = f (f (f (x)) + f (f (x)), then the greatest integer less than or equal to g(1) is___
[25-Jun-2022-Shift-1]

Answer: 2

Solution:

Given,

$$f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)(2+x^{25})\right)^{\frac{1}{50}}$$

and $g(x)=f(f(f(x)))+f(f(x))$

$\therefore g(1)=f(f(f(1)))+f(f(1))$

Now, $f(1)=\left(2\left(1-\frac{1^{25}}{2}\right)(2+1^{25})\right)^{\frac{1}{50}}$

$$=\left(2\left(1-\frac{1}{2}\right)(2+1)\right)^{\frac{1}{50}}$$

$$= (3)^{\frac{1}{50}}$$

$$\therefore f(f(1)) = f\left(3^{\frac{1}{50}}\right)$$

$$= \left(2\left(1 - \frac{\left(3^{\frac{1}{50}}\right)^{25}}{2}\right)\left(2 + \left(3^{\frac{1}{50}}\right)^{25}\right)\right)^{\frac{1}{50}}$$

$$= \left(2\left(1 - \frac{3^{\frac{1}{2}}}{2}\right)\left(2 + 3^{\frac{1}{2}}\right)\right)^{\frac{1}{50}}$$

$$= \left(2 \times \left(\frac{2 - \sqrt{3}}{2}\right)(2 + \sqrt{3})\right)^{\frac{1}{50}}$$

$$= [(2 - \sqrt{3})(2 + \sqrt{3})]^{\frac{1}{50}}$$

$$= (4 - 3)^{\frac{1}{50}}$$

$$= 1^{\frac{1}{50}} = 1$$

Now, $f(f(f(1))) = f(1) = 3^{\frac{1}{50}}$

$$\therefore g(1) = f(f(f(1))) + f(f(1))$$

$$= 3^{\frac{1}{50}} + 1$$

Now, greatest integer less than or equal to $g(1)$

$$= [g(1)]$$

$$= \left[3^{\frac{1}{50}} + 1\right]$$

$$= \left[3^{\frac{1}{50}}\right] + [1]$$

$$= [1.02] + 1$$

$$= 1 + 1 = 2$$

Question88

Let $f(x) = \frac{x-1}{x+1}$, $x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^{n}(x))$ for all $n \in N$, then $f^{6}(6) + f^{7}(7)$ is equal to :
[26-Jun-2022-Shift-1]

Options:

A. $\frac{7}{6}$

B. $-\frac{3}{2}$

C. $\frac{7}{12}$

D. $-\frac{11}{12}$

Answer: B

Solution:

Given,

$$f(x) = \frac{x-1}{x+1}$$

Also given,

$$f^{n+1}(x) = f(f^{n}(x)) \ldots (1)$$

$\therefore$ For $n = 1$

$$f^{1+1}(x) = f(f^{1}(x))$$

$$\Rightarrow f^{2}(x) = f(f(x))$$

$$= f\left(\frac{x-1}{x+1}\right)$$

$$= \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$$

$$= \frac{\frac{x-1-x-1}{x+1}}{\frac{x-1+x+1}{x+1}}$$

$$= \frac{-2}{2x} = -\frac{1}{x}$$

From equation (1), when $n = 2$ $f^{2+1}(x) = f(f^{2}(x))$

$$\Rightarrow f^3(x) = f(f^2(x))$$

$$= f\left(-\frac{1}{x}\right)$$

$$= \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1}$$

$$= \frac{\frac{-1-x}{-1}}{\frac{-1+x}{x}}$$

$$= \frac{-1-x}{-1+x} = \frac{-(x+1)}{x-1}$$

Similarly,

$$f^4(x) = f(f^3(x))$$

$$= f\left(\frac{-x+1}{x-1}\right)$$

$$= \frac{\frac{-(x+1)}{x-1} - 1}{\frac{-(x+1)}{x-1} + 1}$$

$$= \frac{\frac{x-1-x+1}{x-1}}{\frac{-x-1+x-1}{x-1}}$$

$$= \frac{-2x}{-2} = x$$

$$\therefore f^5(x) = f(f^4(x))$$

$$= f(x)$$

$$= \frac{x-1}{x+1}$$

$$f^6(x) = f(f^5(x))$$

$$= f\left(\frac{x-1}{x+1}\right)$$

$= -\frac{1}{x}$ (Already calculated earlier)

$f^7(x) = f(f^6(x))$

$= f\left(-\frac{1}{x}\right)$

$= \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1}$

$= \frac{-(x+1)}{x-1}$

$\therefore f^6(6) = -\frac{1}{6}$

and $f^7(7) = \frac{-(7+1)}{7-1} = -\frac{8}{6}$

So, $f^6(6) + f^7(7)$

$= -\frac{1}{6} - \frac{8}{6}$

$= -\frac{3}{2}$

Question89

The range of the function,

$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$

is

[2021, 01 Sep. Shift-II]

Options:

A. $(0, \sqrt{5})$

B. $[-2, 2]$

C. $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

D. $[0, 2]$

Answer: D

Solution:

Solution:

$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right)\right.$

$\left. + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$

$= \log_{\sqrt{5}}(3 - \sqrt{2}\sin x + \sqrt{2}\cos x)$

$\because -2 \le -\sqrt{2}\sin x + \sqrt{2}\cos x \le 2$

$\Rightarrow 1 \le 3 - \sqrt{2}\sin x + \sqrt{2}\cos x \le 5$

$\Rightarrow \log_{\sqrt{5}} 1 \le \log_{\sqrt{5}}(3 - \sqrt{2}\sin x + \sqrt{2}\cos x)$

$\Rightarrow 0 \le f(x) \le 2$

$\Rightarrow f(x) \in [0, 2]$

Question90

The domain of the function $f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ is [2021, 31 Aug. Shift-II]

Options:

A. $\left[0, \frac{1}{4}\right]$

B. $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

C. $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

D. $\left[0, \frac{1}{2}\right]$

Answer: C

Solution:

$$f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$$

$$-1 \leq \frac{x-1}{x+1} \leq 1 \Rightarrow -1 - 1 \leq \frac{x-1}{x+1} - 1 \leq 1 - 1$$

$$\Rightarrow \ -2 \leq \frac{-2}{x+1} \leq 0 \Rightarrow 0 \leq \frac{1}{x+1} \leq 1$$

$\Rightarrow \ 1 \leq x + 1 < \infty$
$\Rightarrow \ 0 \leq x < \infty$
$\Rightarrow \ x \in [0, \infty)$

and $-1 \leq \dfrac{3x^2 + x - 1}{(x-1)^2} \leq 1$

$\Rightarrow \ -(x-1)^2 \leq 3x^2 + x - 1 \leq (x-1)^2, x \neq 1$
$\Rightarrow \ -(x^2 - 2x + 1) \leq 3x^2 + x - 1$
and $3x^2 + x - 1 \leq x^2 - 2x + 1$
$\Rightarrow \ 4x^2 - x \geq 0$
and $2x^2 + 3x - 2 \leq 0$
$\Rightarrow \ x(4x - 1) \geq 0$
and $(x + 2)(2x - 1) \leq 0$

$\Rightarrow \ x \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty\right)$

and $x \in \left[-2, \frac{1}{2}\right]$

$\Rightarrow \ x \in (-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

Domain of f in Eq. (i) ∩ Eq. (ii)

$\therefore \ x \in \{0\} \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

Question91

Let $f : N \to N$ be a function such that $f(m + n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to

[2021, 31 Aug. Shift-11]

Options:

A. 6

B. 54

C. 18

D. 36

Answer: B

Solution:

$f(m+n) = f(m) + f(n),\ m, n \in N$
$\therefore\ f(3+3) = f(3) + f(3)$
$\Rightarrow\ f(6) = 2f(3) = 18\ \ [\because f(6) = 18]$
$\text{Also}\ \ f(3) = f(2+1) = f(2) + f(1)$
$= f(1+1) + f(1)$
$f(3) = f(1) + f(1) + f(1)$
$\Rightarrow\ 9 = 3f(1) \Rightarrow f(1) = 3$
$\therefore\ f(2) = f(1+1) = f(1) + f(1) = 3 + 3 = 6$
$\text{Hence,}\ \ f(2) \cdot f(3) = 6 \cdot 9 = 54$

Question92

The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is
[2021, 26 Aug. Shift-II]

Options:

A. $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$

B. $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

C. $\left(-\frac{1}{2}, \infty\right) - \{0\}$

D. $\left[-\frac{1}{2}, \infty\right) - \{0\}$

Answer: D

Solution:

$f(x) = \operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)\ \ \left|\frac{1+x}{x}\right| \geq 1$
Clearly, $x \neq 0$
$r\ |1+x|^2 \geq |x|^2$

$1 + x^2 + 2x \geq x^2$
$2x + 1 \geq 0$
$x \geq -\frac{1}{2}$
So,
$x \in \left[-\frac{1}{2}, \infty\right] - \{0\}$

Question93

Which of the following is not correct for relation R on the set of real numbers ?
[2021, 31 Aug. Shift-1]

Options:

A. $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

B. $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

C. $(x, y) \in R \Leftrightarrow x| - |y| \leq 1$ is reflexive but not symmetric.

D. $(x, y) \in R \Leftrightarrow x - y| \leq 1$ is reflexive and symmetric.

Answer: B

Solution:

Solution:
According to the question, let's consider option (b) (2, 3) and (3, 4) satisfy $0 < |x - y| \text{leq} 1$ but (2, 4) does not satisfy it.

Question94

Let N be the set of natural numbers and a relation R on N be defined by $R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is
[2021, 27 July Shift-11]

Options:

A. symmetric but neither reflexive nor transitive.

B. reflexive but neither symmetric nor transitive.

C. reflexive and symmetric, but not transitive.

D. an equivalence relation.

Answer: B

Solution:

Solution:
Given, relation R on N is defined by
$R = \{(x, y) \in N \times N : x^3 - 3x^2 - xy^2 + 3y^3 = 0\}$
$x^3 - 3x^2y - xy^2 + 3y^3 = 0$
$\Rightarrow x^3 - xy^2 - 3x^2y + 3y^3 = 0$
$\Rightarrow x(x^2 - y^2) - 3y(x^2 - y^2) = 0$
$\Rightarrow (x - 3y)(x^2 - y^2) = 0$
$\Rightarrow (x - 3y)(x - y)(x + y) = 0$
Now, $x - x = 0$
$\Rightarrow x = x, \forall (x, x) \in N \times N$
So, R is a reflexive relation.
But not symmetric and transitive relation because,
(3, 1) satisfies but (1, 3) does not. Also, (3, 1) and
(1, −1) satisfies but (3, −1) does not.
Hence, relation R is reflexive but neither symmetric nor transitive.

Question95

Define a relation R over a class of n × n real matrices A and B as "ARB, if there exists a non-singular matrix P such that $PAP^{-1} = B'$. Then which of the following is true ?
[2021, 18 March Shift-II]

Options:

A. R is symmetric, transitive but not reflexive.

B. R is reflexive, symmetric but not transitive.

C. R is an equivalence relation.

D. R is reflexive, transitive but not symmetric.

Answer: C

Solution:

Solution:
For reflexive relation, $\forall (A, A) \in R$ for matrix P.
$\Rightarrow A = PAP^{-1}$ is true for $P = 1$
So, R is reflexive relation.
For symmetric relation,
Let $(A, B) \in R$ for matrix P.
$\Rightarrow A = PBP^{-1}$ After pre-multiply by P^{-1} and post-multiply by P_1
we get
$P^{-1}AP = B$
So, $(B, A) \in R$ for matrix P^{-1}.
So, R is a symmetric relation.
For transitive relation,
Let ARB and BRC
So, $A = PBP^{-1}$ and $B = PCP^{-1}$
Now, $A = P(PCP^{-1})P^{-1}$
$\Rightarrow A = (P)^2C(P^{-1})^2 \Rightarrow A = (P)^2 \cdot C \cdot (P^2)^{-1}$
$\therefore (A, C) \in R$ for matrix P^2.
∴R is transitive relation.
Hence, R is an equivalence relation.

Question96

Let A = {2, 3, 4, 5, ..., 30} and ' ˝ be an equivalence relation on A × A, defined by (a, b) ~ (c, d), if and only if ad = bc. Then, the number of ordered pairs, which satisfy this equivalence relation with ordered pair (4, 3) is equal to
[2021, 16 March Shift-II]

Options:

A. 5

B. 6

C. 8

D. 7

Answer: D

Solution:

Solution:
A = {2, 3, 4, 5, ..., 30}
a = bc
∴ (a, b)R(4, 3)
⇒ 3a = 4b
⇒ $a = \left(\frac{4}{3}\right)b$
b must be a multiple of 3, b can be
(3, 6, 9, . . . 30).
Also, a must be less than or equal to 30.
(a, b) = (4, 3), (8, 6), (12, 9), (16, 12), (20, 15)
(24, 18), (28, 21)
⇒7 ordered pairs

Question97

Let R = { (P, O)|, P and Q are at the same distance from the origin\} be a relation, then the equivalence class of (1, −1) is the set
[2021, 26 Feb. Shift-1]

Options:

A. $S = \{(x, y) \mid x^2 + y^2 = 4\}$

B. $S = \{(x, y) \mid x^2 + y^2 = 1\}$

C. $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

D. $S = \{(x, y) \mid x^2 + y^2 = 2\}$

Answer: D

Solution:

Solution:
Let P(a, b) and Q(c, d) are any two points.
Given, OP = 00
i.e. $\sqrt{a^2+b^2} = \sqrt{c^2+d^2}$
Squaring on both sides,
$R = \{((a, b), (c, d)) : a^2+b^2 = c^2+d^2\}$
$R(x, y)$, $S(1, -1)$, $OR = OS$ (equivalence class)
This gives $OR = \sqrt{x^2+y^2}$ and $OS = \sqrt{2}$
$l \Rightarrow \sqrt{x^2+y^2} = \sqrt{2}$
$\Rightarrow x^2+y^2 = 2($ Squaring on both sides)
$\therefore S = \{(x, y) : x^2+y^2 = 2\}$

Question98

Let {S = 1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to ....
[2021, 27 July Shift-I]

Answer: 490

Solution:

Solution:
$S = \{1, 2, 3, 4, 5, 6, 7\}$
$f : S \to S$
$f(m \cdot n) = f(m)f(n)$
$m, n \in S \Rightarrow m, n \in S$
If $mn \in S \Rightarrow mn \leq 7$
So, $(1 \cdot 1, 1 \cdot 2, \ldots, 1 \cdot 7) \leq 7$
$(2 \cdot 2, 2 \cdot 3) \leq 7$
When $m = 1$, $f(n) = f(1) \cdot f(n) \Rightarrow f(1) = 1$
When $m = n = 2$,
$$f(4) = f(2)f(2) = \begin{cases} f(2) = 1 \Rightarrow f(4) = 1 \text{ or} \\ f(2) = 2 \Rightarrow f(4) = 4. \end{cases}$$
When, $m = 2$, $n = 3$
$$f(6) = f(2)f(3) \begin{cases} \text{When, } f(2) = 1 \\ f(3) = 1 \text{ to } 7 \\ \text{When, } f(2) = 2 \\ f(3) = 1 \text{ or } 2 \text{ or } 3. \end{cases}$$
And $f(5)$, $f(7)$ can take any value (1-7) [$\because f(5) = f(1) \cdot f(5) \leq 7$ and $f(7) = f(1) \cdot f(7) \leq 7$ } The possible combination is
$l \, l \, f(1) = 1 \qquad f(1) = 1$
$f(2) = 1 \qquad f(2) = 2$
$f(3) = (1-7) \qquad f(3) = (1-3)$
$f(4) = 1 \qquad f(4) = 4$
$f(5) = (1-7) \qquad f(5) = (1-7)$
$f(6) = f(3) \qquad f(6) = f(3)$
$f(7) = (1-7) \qquad f(7) = (1-7)$
So, total $= (1 \times 1 \times 7 \times 1 \times 7 \times 1 \times 7)$
$+(1 \times 1 \times 3 \times 1 \times 7 \times 1 \times 7)$
$= 490$

Question99

If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to
[25 July 2021, Shift-III]

Options:

A. 0

B. 4

C. -2

D. 2

Answer: D

Solution:

Solution:
We have,
$\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ (∵[x] is the greatest integer function)
Substitute the values of n

$$\begin{aligned} &= [4]+[-4.5]+[5]+[-5.5] \\ &\quad +\ldots+[-49.5]+[50] \\ &= 4-5+5-6+\ldots-50+50 \\ &= 4 \end{aligned}$$

Question100

If the domain of the function $f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval (α, β), then α + β is equal to
[2021, 22 July Shift-II]

Options:

A. $\frac{3}{2}$

B. 2

C. $\frac{1}{2}$

D. 1

Answer: A

Solution:

Solution:

$f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\dfrac{2x-1}{2}\right)}}$

$\Rightarrow x \in R,\ \ x(x-1) \le 0$

$x^2 - x + 1 \ge 0$ and $x^2 - x + 1 \le 1$

$0 \le x \le 1 \ \ \cdots\cdots (i) \Rightarrow \ 0 < \sin^{-1}\left(\dfrac{2x-1}{2}\right) < \dfrac{\pi}{2}$

$\Rightarrow \ 0 < \dfrac{2x-1}{2} < 1$

$\Rightarrow \ \dfrac{1}{2} < x < \dfrac{3}{2} \ \ \cdots\cdots (ii)$

$(A) \cap (B) \ = x \in \left(\dfrac{1}{2}, 1\right]$

$\therefore \ \alpha + \beta \ = \dfrac{3}{2}$

Question101

Let [x] denote the greatest integer ≤x, where x ∈ R. If the domain of the real valued function

$f(x) = \sqrt{\dfrac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c)$

$\cup [u, \infty)$, $a < b < c$, then the value of
$a + b + c$ is
[2021, 20 July Shift-I]

Options:

A. 8

B. 1

C. −2

D. −3

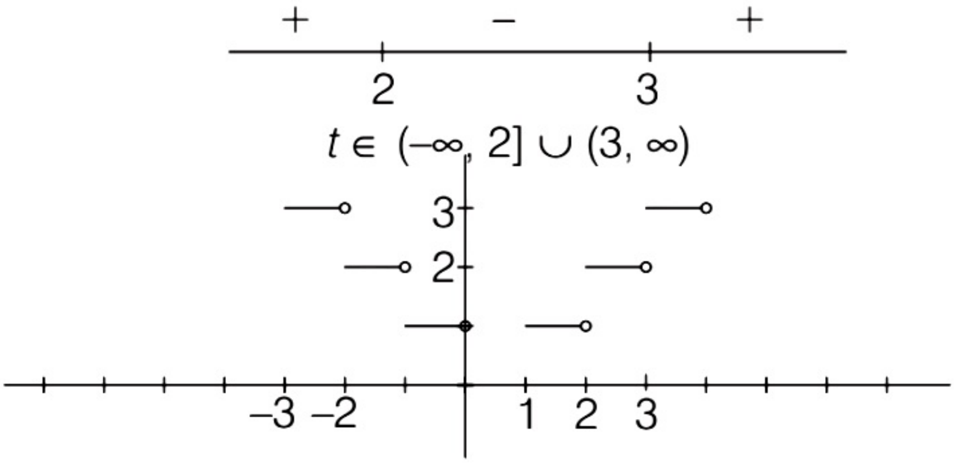
Answer: C

Solution:

Solution:

$f(x) = \sqrt{\dfrac{|[x]|-2}{|[x]|-3}}\ \dfrac{|[x]|-2}{|[x]|-3} \ge 0$

Let $|[x]| = t$



l $|[x]| = 3 \Rightarrow x \in [-3, -2) \cup [3, 4)$

Domain of $x = [-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$a = -3$

$b = -2$

$c = 3$

$\therefore\ a + b + c = -3 + (-2) + 3 = -2$

Question102

The real valued function $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to [2021, 18 March Shift-I]

Options:

A. all reals except integers

B. all non-integers except the interval $[-1, 1]$

C. all integers except $0, -1, 1$

D. all reals except the interval $[-1, 1]$

Answer: B

Solution:

Solution:

Given, $f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{x - [x]}}$

$\Rightarrow f(x) = \frac{\operatorname{cosec}^{-1} x}{\sqrt{\{x\}}}$

For $f(x)$ to be defined,

$$\begin{cases} |x| \geq 1 \\ \{x\} > 0. \end{cases} \Rightarrow \begin{cases} x \leq -1 \text{ or } x \geq 1 \\ x \neq 1 \text{ integers} \end{cases}.$$

i.e. $x \in (-\infty, -1] \cup [1, \infty) - \{\text{integers}\}$

i.e. all non-integers except the interval $[-1, 1]$
(here, -1 and 1 are included in except case, because of -1 and 1 are integers).

Question103

If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions?
$f+g, f-g, f/g, g/f, g-f$, where
$(f \pm g)(x) = f(x) \pm g(x), (f/g)(x) = \frac{f(x)}{g(x)}$
[2021, 18 March, Shift-1]

Options:

A. $0 \le x \le 1$

B. $0 \le x < 1$

C. $0 < x < 1$

D. $0 < x \le 1$

Answer: C

Solution:

Solution:
Given, $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$
$\therefore$ Domain of $f(x) = D_1$ is $x \ge 0$
i.e. $D_1 : x \in (0, \infty)$
and domain of $g(x) = D_2$ is $1 - x \ge 0 \Rightarrow x \le 1$
i.e. $D_2 : x \in (-\infty 1]$
As, we know that, the domain of $f + g_1 f - g, g - f$ will be $D_1 \cap D_2$ as well as the domain for $\frac{f}{g}$ is $D_1 \cap D_2$ except all those value(s) of x, such that $g(x) = 0$.
Similarly, for $\frac{g}{f}$ is $D_1 \cap D_2$ but $f(x) \ne 0$.
Hence, common domain for $(f+g)$, $(f-g)$, $\left(\frac{f}{g}\right)$, $\left(\frac{g}{f}\right)$ and $(g-f)$ will be $0 < x < 1$

Question104

A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series
$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots + f\left(\frac{39}{20}\right)$
is equal to
[2021, 25 Feb. Shift-II]

Options:

A. $\frac{29}{2}$

B. $\frac{49}{2}$

C. $\frac{39}{2}$

D. $\frac{19}{2}$

Answer: C

Solution:

Solution:

Given, $f(x) = \frac{5^x}{5^x + 5}$, then,

$f(2 - x) = \frac{5^{2-x}}{5^{2-x} + 5}$

$= \frac{5}{5^x + 5}$

This gives, $f(x) + f(2 - x) = \frac{5^x + 5}{5^x + 5} = 1 \Rightarrow f\left(\frac{1}{20}\right) + f\left(2 - \frac{1}{20}\right) = f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right) = 1$

Similarly,

$cf\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right) = 1$ and so on,

$\therefore f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + \ldots + f\left(\frac{38}{20}\right) + f\left(\frac{39}{20}\right)$

$= 1 + 1 + \ldots + 1 + f\left(\frac{20}{20}\right)$

$= 19 + f(1) = 19 + \frac{1}{2} = \frac{39}{2}$

Question105

If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value of expression $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$ is

[2021, 24 Feb. Shift-II]

Answer: 2

Solution:

Solution:

Given, $a + \alpha = 1$

$b + \beta = 2$

$\because a \cdot f(x) + \alpha \cdot f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ······ (i)

Replace x by $\frac{1}{x}$,

$af\left(\frac{1}{x}\right) + af(x) = \frac{b}{x} + \beta x$

Adding Eqs. (i) and (ii), we get

$(a+\alpha)\left[f(x)+f\left(\frac{1}{x}\right)\right]=\left(x+\frac{1}{x}\right)(b+\beta)$ .... (ii)

$\Rightarrow \frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=\frac{b+\beta}{a+\alpha}=\frac{2}{1}=2$

Question106

Let $f(x)=\sin^{-1}x$ and
$g(x)=\frac{x^2-x-2}{2x^2-x-6}$
If $g(2)=\lim_{x\to 2} g(x)$, then the domain of thefunction fog is
[2021, 26 Feb. Shift-II]

Options:

A. $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

B. $(-\infty,-2]\cup[-1,\infty)$

C. $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

D. $(-\infty,-1]\cup[2,\infty)$

Answer: C

Solution:

Solution:

Given, $g(x)=\frac{x^2-x-2}{2x^2-x-6}$, $f(x)=\sin^{-1}x$

$f(g(x))=\sin^{-1}(g(x))$

$f\circ g(x)=\sin^{-1}\left(\frac{x^2-x-2}{2x^2-x-6}\right)$

For the domain of $f\,og(x)$,

$|g(x)|\le 1$

[∵ Domain of $f(x)$ is $[-1,1]$]

$\Rightarrow \left|\frac{x^2-x-2}{2x^2-x-6}\right|\le 1$

$\Rightarrow \left|\frac{(x+1)(x-2)}{(2x+3)(x-2)}\right|\le 1$

$\Rightarrow \left|\frac{x+1}{2x+3}\right|\le 1$

$\Rightarrow -1\le\frac{x+1}{2x+3}\le 1$

$\Rightarrow \left(\frac{x+1}{2x+3}\right)^2\le 1$

$\Rightarrow (x+1)^2\le(2x+3)^2$

$\Rightarrow 3x^2+10x+8\ge 0$

$\Rightarrow\ (3x + y)(x + 2) \geq 0$
This implies,
$x \in (-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

Question107

Let $g : N \rightarrow N$ be defined as
$g(3n + 1) = 3n + 2$
$g(3n + 2) = 3n + 3,$
$g(3n + 3) = 3n + 1,$ for all $n \geq 0.$
Then which of the following statements is true?
[2021, 25 July Shift-1]

Options:

A. There exists an onto function $f : N \rightarrow N$ such that $fog = f$

B. There exists a one-one function $f : N \rightarrow N$ such that $fog = f$

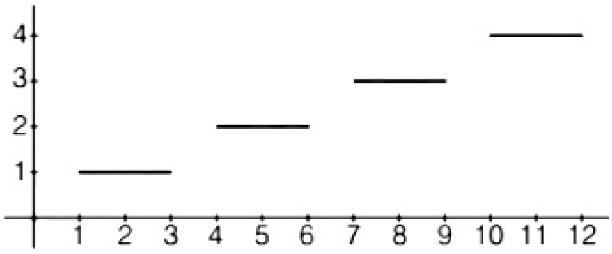
C. $gogog = g$

D. There exists a function $f : N \rightarrow N$ such that $gof = f$

Answer: A

Solution:

$g(3n + 1) = 3n + 2$
$g(3n + 2) = 3n + 3$
$g(3n + 3) = 3n + 1$, for all $n \geq 0$
$g : N \rightarrow N$
$g(1) = 2, g(4) = 5, g(7) = 8$
$g(2) = 3, g(5) = 6, g(8) = 9$
$g(3) = 1, g(6) = 4, g(9) = 7$
$\Rightarrow\ f[g(1)] = f(1)$
$\Rightarrow\ f(2) = f(1)$
Clearly, it is not a one - one function.
Now, $f[g(2)] = f(2)$
$f(3) = f(2)$
And, $f[g(3)] = f(3)$
$f(1) = f(3)$
Similarly, $f[g(4)] = f(4)$
$f(5) = f(4)$
And, so on
l $f(1) = f(2) = f(3)$
$f(4) = f(5) = f(6)$



Now, there can be a possibility such that
So, $f(x)$ can be onto function.
When $f(1) = f(2) = f(3) = 1$
$f(4) = f(5) = f(6) = 2$
and so on.

Question108

Consider function $f: A \rightarrow B$ and $g: B \longrightarrow C(A, B, C \subseteq R)$ such that $(gof)^{-1}$ exists, then
[2021, 25 July Shift-II]

Options:

A. f and g both are one-one

B. f and g both are onto

C. f is one-one and g is onto

D. f is onto and g is one-one

Answer: C

Solution:

Solution:
Given functions, $f: A \rightarrow B$ and $g: B \rightarrow C(A, B, C \subseteq R)$
$\therefore (gof)^{-1}$ exists $\Rightarrow$ gof is a bijective function.
$\Rightarrow$ ' f ' must be 'one-one' and ' g ' must be 'onto' function.

Question109

Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then, the number of bijective functions $F: A \rightarrow A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to ....
[2021, 22 July Shift-III]

Answer: 720

Solution:

Solution:
$f(1) + f(2) = 3 - f(3)$
$A = \{0, 1, 2, 3, 4, 5, 6, 7\}$
$f: A \rightarrow A$
So, $f(1) + f(2) + f(3) = 3$
$0 + 1 + 2 = 3$ is the only possibility.
So, $f(0)$ can be either 0 or 1 or 2 .
Similarly, $f(1)$ and $f(2)$ can be 0,1 and 2 .
and $\{3, 4, 5, 6, 7\} \longrightarrow \{3, 4, 5, 6, 7\}$

They have 5 ! choices.
And $\{0, 1, 2\}$

They have 3! choices.
Number of bijective functions
$= 3! \times 5! = 720$

Question110

Let $f : R - \left\{ \frac{\alpha}{6} \right\} \longrightarrow R$ be defined by
$f(x) = \frac{5x + 3}{6x - \alpha}$
Then, the value of α for which $(fof)(x) = x$, for all $x \in R - \left\{ \frac{\alpha}{6} \right\}$ is
[2021, 20 July Shift-II]

Options:

A. No such α exists

B. 5

C. 8

D. 6

Answer: B

Solution:

Solution:
$f(x) = \frac{5x + 3}{6x - \alpha}$
Now, $f \circ f(x) = f\left(\frac{5x + 3}{6x - \alpha}\right)$
$= \frac{5\left(\frac{5x + 3}{6x - \alpha}\right) + 3}{6\left(\frac{5x + 3}{6x - \alpha}\right) - \alpha}$
$= \frac{5(5x + 3) + 3(6x - \alpha)}{6(5x + 3) - \alpha(6x - 2)}$
$= \frac{5(5x + 3) + 3(6x - \alpha)}{6(5x + 3) - \alpha(6x - 2)}$
Given, $fof(x) = x$
$\Rightarrow \frac{5(5x + 3) + 3(6x - \alpha)}{6(5x + 3) - \alpha(6x - \alpha)} = x$
$\Rightarrow 25x + 15 + 18x - 3\alpha$
$= 30x^2 + 18x - 6\alpha x^2 + \alpha^2 x$
$\Rightarrow x^2(30 - 6\alpha) - x(\alpha^2 - 25) + 3\alpha - 15 = 0$
Comparing coefficients,
$30 - 6x = 0$
$\Rightarrow 6\alpha = 30$
$\Rightarrow \alpha = 5$

Question111

Let $f : R - \{3\} \rightarrow R - \{1\}$ be defined by $f(x) = \frac{x - 2}{x - 3}$. Let $g : R \rightarrow R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to
[2021, 18 March Shift-II]

Options:

A. 7

B. 2

C. 5

D. 3

Answer: C

Solution:

Solution:

Given, $f(x) = \frac{x-2}{x-3}$

$g(x) = 2x - 3$

Let $y = f(x) = \frac{x-2}{x-3}$

$\Rightarrow xy - 3y = x - 2 \Rightarrow xy - x = 3y - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\Rightarrow f^{-1}(y) = \frac{3y-2}{y-1}$

$\Rightarrow f^{-1}(x) = \frac{3x-2}{x-1}$

Similarly, $g^{-1}(x) = \frac{x+3}{2}$

Given, $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$

$\Rightarrow x^2 + 8x - 7 = 13(x-1)$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x = 2, 3$

$\therefore$ Sum $= 2 + 3 = 5$

Question112

The inverse of $y = 5^{\log x}$ is
[2021, 17 March Shift-I]

Options:

A. $x = 5^{\log y}$

B. $x = y^{\log 5}$

C. $x = y^{\frac{1}{\log 5}}$

D. $x = 5^{\frac{1}{\log y}}$

Answer: C

Solution:

Solution:
$y = 5^{\log x}$
Taking log on both sides,
$\Rightarrow \ \log y = \log x \cdot \log 5$
$\Rightarrow \ \frac{1}{\log 5} = \frac{\log x}{\log y}$
$\frac{1}{\log 5} = \log_y x$
$x = y^{\frac{1}{\log 5}}$

Question113

Let A = {1, 2, 3, ..., 10} and f : A → A be defined as defined as

$$\mathbf{f(x)} = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x & \text{if } x \text{ is even} \end{cases}.$$

Then, the number of possible functions g : A → A, such that gof = f is [2021, 26 Feb. Shift-II]

Options:

A. 10^5

B. ${}^{10}C_5$

C. 5^5

D. 5 !

Answer: A

Solution:

Solution:

$$f(x) = \begin{cases} x+1 & x \text{is odd} \\ x & x \text{is even.} \end{cases}.$$

Given, $g : A \to A$ such that,
$g(f(x)) = f(x)$
When x is even, then
$g(x) = x$
When x is odd, then
$g(x+1) = x+1$
This implies,
$g(x) = x_1 x$ is even.
$\Rightarrow$ If x is odd, then $g(x)$ can take any value in set A.
So, number of $g(x) = 10^5$

Question114

Let f , g : N → N , such that f (n + 1) = f (n) + f (1) ∀n ∈ N and g be any

arbitrary function. Which of the following statements is not true? [2021, 25 Feb. Shift-1]

Options:

A. if $f \circ g$ is one-one, then g is one-one.

B. if f is onto, then $f(n) = n, \forall n \in N$.

C. f is one-one.

D. if g is onto, then fog is one-one.

Answer: D

Solution:

Solution:
Given, $f(n+1) = f(n) + f(1), \forall n \in N$
$\Rightarrow f(n+1) - f(n) = f(1)$
It is an AP with common difference $= f(1)$
Also, general term
$11 = T_n = f(1) + (n-)f(1) = nf(1)$
$\Rightarrow f(n) = nf(1)$
Clearly, $f(n)$ is one-one.
For fog to be one-one, g must be one-one.
For f to be onto, $f(n)$ should take all the values of natural numbers.
As, $f(x)$ is increasing, $f(1) = 1$
$\Rightarrow f(n) = n$
If g is many-one, then $f \circ g$ is many one.
So, if g is onto, then fog is one-one.

Question115

Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then, [2021, 25 Feb. Shift-II]

Options:

A. $2y = 91x$

B. $2y = 273x$

C. $y = 91x$

D. $y = 273x$

Answer: A

Solution:

$x = \{ f : A \to B, f \text{ is one - one} \}$
$y = \{ g : A \to A \times B, g \text{ is one one} \}$

Number of elements in A = 3 i.e. |A| = 3
Similarly, |B| = 5
Then, $|A \times B| = |A| \times |B| = 3 \times 5 = 15$
Now, number of one-one function from A to B will be
$l\ {}^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$
$\therefore\ x = 60$
Now, number of one-one function from A
l to $A \times B$ will be $= {}^{15}P_{3} = \frac{15!}{(15-3)!} = \frac{15!}{12!}$
$= 15 \times 14 \times 13 = 2730$
$\therefore\ y = 2730\ l\ c\ \therefore\ y = 2730$
Thus, $2 \times (2730) = 91 \times (60)$
$\Rightarrow 2y = 91x$

Question116

Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is :
24 Feb 2021 Shift 1

Options:

A. onto but not one-one

B. both one-one and onto

C. one-one but not onto

D. neither one-one nor onto

Answer: C

Solution:

Solution:
$f(g(x)) = 2g(x) - 1 = 2\left(\frac{2x-1}{2(x-1)}\right) - 1 = \frac{x}{x-1} = 1 + \frac{1}{x-1}$
Range of $f(g(x)) = \text{mathbb}\,R - \{1\}$
Range of $f(g(x))$ is not onto
&$f(g(x))$ is one-one
So, $f(g(x))$ is one-one but not onto.

Question117

Let R_1 and R_2 be two relations defined as follows :
$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and $R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$, where Q is the set of all rational numbers. Then :
[Sep. 03, 2020 (II)]

Options:

A. Neither R_1 nor R_2 is transitive.

B. R_2 is transitive but R_1 is not transitive.

C. R_1 is transitive but R_2 is not transitive.

D. R_1 and R_2 are both transitive.

Answer: A

Solution:

Solution:
(a) For R_1 let $a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = 8^{1/4}$
$aR_1b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$
$bR_1c \Rightarrow b^2 + c^2 = (1 - \sqrt{2})^2 + (8^{1/4})^2 = 3 \in Q$
$aR_1c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (8^{1/4})^2 = 3 + 4\sqrt{2} \notin Q$
$\therefore R_1$ is not transitive.
For R_2 let $a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$
$aR_2b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (\sqrt{2})^2 = 5 + 2\sqrt{2} \notin Q$
$bR_2c \Rightarrow b^2 + c^2 = (\sqrt{2})^2 + (1 - \sqrt{2})^2 = 5 - 2\sqrt{2} \notin Q$
$aR_2c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$
$\therefore R_2$ is not transitive.

Question118

The domain of the function $f(x) = \sin^{-1}\left(\frac{|x| + 5}{x^2 + 1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to :
[Sep. 02, 2020 (I)]

Options:

A. $\frac{\sqrt{17}}{2}$

B. $\frac{\sqrt{17} - 1}{2}$

C. $\frac{1 + \sqrt{17}}{2}$

D. $\frac{\sqrt{17}}{2} + 1$

Answer: C

Solution:

Solution:
$\because f(x) = \sin^{-1}\left(\frac{|x| + 5}{x^2 + 1}\right)$
$\therefore -1 \le \frac{|x| + 5}{x^2 + 1} \le 1$
$\Rightarrow |x| + 5 \le x^2 + 1$
$[\because x^2 + 1 \ne 0]$
$\Rightarrow x^2 - |x| - 4 \ge 0$

$\Rightarrow \left(|x| - \frac{1-\sqrt{17}}{2} \right) \left(|x| - \frac{1+\sqrt{17}}{2} \right) \geq 0$

$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2} \right) \cup \left[\frac{1+\sqrt{17}}{2}, \infty \right)$

$\therefore a = \frac{1+\sqrt{17}}{2}$

Question119

If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :
[Sep. 02, 2020 (I)]

Options:

A. $\{-2, -1, 1, 2\}$

B. $\{0, 1\}$

C. $\{-2, -1, 0, 1, 2\}$

D. $\{-1, 0, 1\}$

Answer: D

Solution:

Solution:

Since, $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$

$\therefore R = \{(1, 1), (2, 1), (1, -1), (0, 1), (1, 0)\}$

$\Rightarrow D_{R^{-1}} = \{-1, 0, 1\}$

Question120

Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x, $[x]^2 + 2[x+2] - 7 = 0$ has :
[Sep. 04, 2020 (I)]

Options:

A. exactly two solutions

B. exactly four integral solutions

C. no integral solution

D. infinitely many solutions

Answer: D

Solution:

Solution:

The given equation $[x]^2 + 2[x] + 4 - 7 = 0$
$\Rightarrow [x]^2 + 2[x] - 3 = 0$
$\Rightarrow [x]^2 + 3[x] - [x] - 3 = 0$
$\Rightarrow ([x] + 3)([x] - 1) = 0 \Rightarrow [x] = 1$ or -3 $\Rightarrow x \in [-3, -2) \cup [1, 2)$
∴ The equation has infinitely many solutions.

Question121

Let $f(x)$ be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in:
[Sep. 02, 2020 (II)]

Options:

A. (-1,0)

B. (1,3)

C. (-3,-1)

D. (0,1)

Answer: A

Solution:

Solution:
Let $f(x) = ax^2 + bx + c$
Given : $f(-1) + f(2) = 0$
$a - b + c + 4a + 2b + c = 0$
$\Rightarrow 5a + b + 2c = 0$ ......(i)
and $f(3) = 0 \Rightarrow 9a + 3b + c = 0$ ......(ii)
From equations (i) and (ii),

$$\frac{a}{1-6} = \frac{b}{18-5} = \frac{c}{15-9} \Rightarrow \frac{a}{-5} = \frac{b}{13} = \frac{c}{6}$$

Product of roots, $\alpha\beta = \frac{c}{a} = \frac{-6}{5}$ and $\alpha = 3$

$\Rightarrow \beta = \frac{-2}{5} \in (-1, 0)$

Question122

Let $f(1, 3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1 + x^2}$ where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is:
[Jan. 8, 2020 (II)]

Options:

A. $\left(\frac{2}{5}, \frac{3}{5}\right) \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

B. $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right)$

C. $\left(\frac{2}{5}, \frac{4}{5}\right)$

D. $\left(\frac{3}{5}, \frac{4}{5}\right)$

Answer: B

Solution:

Solution:

$$f(x) \begin{cases} \frac{x}{x^2+1} & x \in (1, 2) \\ \frac{2x}{x^2+1} & x \in [2, 3). \end{cases}$$

$$f'(x) \begin{cases} \frac{1-x^2}{1+x^2} & x \in (1, 2) \\ \frac{1-2x^2}{1+x^2} & x \in [2, 3). \end{cases}$$

$\therefore f(x)$ is a decreasing function

$\therefore\ y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{6}{10}, \frac{4}{5}\right]$

$\Rightarrow\ y \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

Question123

Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one $\}$ is ______.
[NA Sep. 05,2020 (II)]

Answer: 19

Solution:

Solution:
The desired functions will contain either one element or two elements in its codomain of which '2' always belongs to $f(A)$.
$\therefore$ The set B can be $\{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}$
Total number of functions $= 1 + (2^3 - 2)3 = 19$

Question124

The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$, is ______.
[Jan. 8, 2020 (I)]

Options:

A. $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

B. $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

C. $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

D. $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

Answer: A

Solution:

Solution:

$y=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$

$\frac{1+y}{1-y}=\frac{8^{2x}}{8^{-2x}}\Rightarrow 8^{4x}=\frac{1+y}{1-y}$

$\Rightarrow 4x=\log_8\left(\frac{1+y}{1-y}\right)$

$\Rightarrow x=\frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)$

$\therefore f^{-1}(x)=\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

Question125

If $g(x)=x^2+x-1$ and $(gof)(x)=4x^2-10x+5$, then $f\left(\frac{5}{4}\right)$ is equal to: [Jan. 7, 2020 (I)]

Options:

A. $\frac{3}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $-\frac{3}{2}$

Answer: B

Solution:

Solution:

$(gof)(x)=g(f(x))=f^2(x)+f(x)-1$

$g\left(f\left(\frac{5}{4}\right)\right)=4\left(\frac{5}{4}\right)^2-10\cdot\frac{5}{4}+5=-\frac{5}{4}$

$[\because g(f(x))=4x^2-10x+5]$

$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$

$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$

$t\left(\frac{5}{4}\right) = -\frac{1}{2}$

Question126

For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to:

[Sep. 06, 2020 (II)]

Options:

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. -3

D. 3

Answer: D

Solution:

Solution:

$$f(f(x)) = \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)} = x$$

$$\Rightarrow \frac{a-ax}{1+x} = f(x) \Rightarrow \frac{a(1-x)}{1+x} = \frac{a-x}{a+x} \Rightarrow a = 1$$

$$\therefore f(x) = \frac{1-x}{1+x} \Rightarrow f\left(-\frac{1}{2}\right) = 3$$

Question127

Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is :

[Jan. 11, 2019 (I)]

Options:

A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. $R - [-1, 1]$

C. $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$

D. $(-1, 1) - \{0\}$

Answer: A

Solution:

Solution:

$f(x) = \frac{x}{1 + x^2}, x \in R$

Let, $y = \frac{x}{1 + x^2}$

$\Rightarrow yx^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1 - 4y^2}}{2}$

$\Rightarrow 1 - 4y^2 \geq 0$

$\Rightarrow 1 \geq 4y^2$

$\Rightarrow |y| \leq \frac{1}{2}$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$

$\Rightarrow$ The range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Question128

The domain of the definition of the function $f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$ is: [April. 09, 2019 (II)]

Options:

A. $(-1,0) \cup (1, 2) \cup (3, \infty)$

B. $(-2,-1) \cup (-1, 0) \cup (2, \infty)$

C. $(-1,0) \cup (1, 2) \cup (2, \infty)$

D. $(1,2) \cup (2, \infty)$

Answer: C

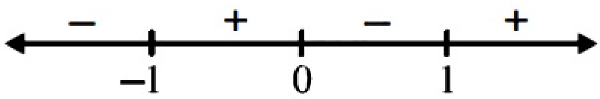
Solution:

Solution:

To determine domain, denominator $\neq 0$ and $x^3 - x > 0$

i.e., $4 - x^2 \neq 0 x \neq \pm 2$ ......(1)

and $x(x - 1)(x + 1) > 0$



$x \in (-1, 0) \cup (1, \infty)$ ....(2)

Hence domain is intersection of (1)&(2).

i.e., $x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

Question129

If $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

[April 8, 2019 (I)]

Options:

A. $2f(x)$

B. $2f(x^2)$

C. $(f(x))^2$

D. $-2f(x)$

Answer: A

Solution:

Solution:

$f(x) = \log\left(\frac{1-x}{1+x}\right), |x| < |$

$$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+2x^2}}\right)$$

$$= \log\left(\frac{1+x^2-2x}{1+x^2+2x}\right) = \log\left(\frac{1-x}{1+x}\right)^2$$

$$= 2\log\left(\frac{1-x}{1+x}\right) = 2f(x)$$

Question130

Let $f(x) = a^x (a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals:

[April. 08, 2019 (II)]

Options:

A. $2f_1(x)f_1(y)$

B. $2f_1(x+y)f_1(x-y)$

C. $2f_1(x)f_2(y)$

D. $2f_1(x+y)f_2(x-y)$

Answer: A

Solution:

Solution:
Given function can be written as

$f(x) = a^x = \left(\frac{a^x + a^{-x}}{2}\right) + \left(\frac{a^x - a^{-x}}{2}\right)$

where $f_1(x) = \frac{a^x + a^{-x}}{2}$ is even function

$f_2(x) = \frac{a^x - a^{-x}}{2}$ is odd function

$\Rightarrow f_1(x+y) + f_1(x-y)$

$= \left(\frac{a^{x+y} + a^{-x-y}}{2}\right) + \left(\frac{a^{x-y} + a^{-x+y}}{2}\right)$

$= \frac{1}{2}[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})]$

$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2} = 2f_1(x) \cdot f_1(y)$

Question131

Let a function $f : (0, \infty) \to (0, \infty)$ be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is : [Jan. 11, 2019 (II)]

Options:

A. not injective but it is surjective

B. injective only

C. neither injective nor surjective

D. (Bonus)

Answer: D

Solution:

Solution:
$f : (0, \infty) \to (0, \infty)$

$f(x) = \left|1 - \frac{1}{x}\right|$ is not a function

$\because f(1) = 0$ and $1 \in$ domain but $0 \notin$ co-domain
Hence, $f(x)$ is not a function.

Question132

The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f (k) is a multiple of 3, whenever k is a multiple of 4 is : [Jan. 11, 2019 (II)]

Options:

A. $6^5 \times (15)!$

B. $5! \times 6!$

C. $(15)! \times 6!$

D. $5^6 \times 15$

Answer: C

Solution:

Solution:
Domain and codomain $= \{1, 2, 3, \ldots\ldots, 20\}$.
There are five multiple of 4 as 4,8,12,16 and 20 .
and there are 6 multiple of 3 as 3,6,9,12,15,18 .
Since, when ever k is multiple of 4 then $f(k)$ is multiple of 3 then total number of arrangement $= {}^6c_5 \times 5! = 6!$
Remaining 15 elements can be arranged in $15!$ ways.
Since, for every input, there is an output
⇒ function $f(k)$ in onto
∴ Total number of arrangement $= 15!.6!$

Question133

Let N be the set of natural numbers and two functionsf and g be defined as f, g : N → N such that $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$

and $g(n) = n - (-1)^n$**. Then f og is:**
[Jan. 10, 2019 (II)]

Options:

A. onto but not one-one.

B. one-one but not onto.

C. both one-one and onto.

D. neither one-one nor onto.

Answer: A

Solution:

Solution:

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$$

$$g(n) = \begin{cases} 2, & n = 1 \\ 1, & n = 2 \\ 4, & n = 3 \\ 3, & n = 4 \\ 6, & n = 5 \\ 5, & n = 6 \end{cases}$$

Then,

$$f(g(n)) = \begin{cases} \frac{g(n)+1}{2}, & \text{if } g(n) \text{ is odd} \\ \frac{g(n)}{2}, & \text{if } g(n) \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} 1, & n = 1 \\ 1, & n = 2 \\ 2, & n = 3 \\ 2, & n = 4 \\ 3, & n = 5 \\ 3, & n = 6 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{cases}$$

$\Rightarrow$ fog is onto but not one - one

Question134

Let $A = \{x \in R : x$ is not a positive integer $\}$. Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is:
[Jan. 09, 2019 (II)]

Options:

A. not injective

B. neither injective nor surjective

C. surjective but not injective

D. injective but not surjective

Answer: D

Solution:

Solution:
As $A = \{x \in R : x$ is not a positive integer $\}$
A function $f : A \to R$ given by $f(x) = \frac{2x}{x-1}$
$f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$
So, f is one-one.
As $f(x) \neq 2$ for any $x \in A \Rightarrow f$ is not onto.
Hence f is injective but not surjective.

Question135

For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$

If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to

[April 12, 2019 (I)]

Options:

A. $\tan\frac{\pi}{12}$

B. $\tan\frac{11\pi}{12}$

C. $\tan\frac{7\pi}{12}$

D. $\tan\frac{5\pi}{12}$

Answer: B

Solution:

Solution:
$\because \phi(x) = ((hof)og)(x)$
$\because \phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right) = h(f(\sqrt{3})) = h(3^{1/4})$
$= \frac{1-\sqrt{3}}{1+\sqrt{3}} = -\frac{1}{2}(1+3-2\sqrt{3}) = \sqrt{3}-2 = -(-\sqrt{3}+2)$
$= -\tan 15° = \tan(180° - 15°) = \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

Question136

Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true? [April 10, 2019 (I)]

Options:

A. $g(f(S)) \neq S$

B. $f(g(S)) = S$

C. $g(f(S)) = g(S)$

D. $f(g(S)) \neq f(S)$

Answer: C

Solution:

Solution:
$f(x) = x^2; x \in R$
$g(A) = \{x \in R : f(x) \in A\} S = [0, 4]$
$g(S) = \{x \in R : f(x) \in S\}$
$= \{x \in R : 0 \le x^2 \le 4\} = \{x \in R : -2 \le x \le 2\}$
$\therefore g(S) \neq S \therefore f(g(S)) \neq f(S)$
$g(f(S)) = \{x \in R : f(x) \in f(S)\}$
$= \{x \in R : x^2 \in S^2\} = \{x \in R : 0 \le x^2 \le 16\}$
$= \{x \in R : -4 \le x \le 4\}$
$\therefore g(f(S)) \neq g(S)$
$\therefore g(f(S)) = g(S)$ is incorrect.

Question137

For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then $J(x)$ is equal to:
[Jan. 09, 2019 (I)]

Options:

A. $f_3(x)$

B. $\frac{1}{x} f_3(x)$

C. $f_2(x)$

D. $f_1(x)$

Answer: A

Solution:

Solution:
The given relation is

$(f_2 o J o f_1)(x) = f_3(x) = \frac{1}{1-x}$

$\Rightarrow (f_2 J)(f_1(x)) = \frac{1}{1-x}$

$\Rightarrow (f_2 o J)\left(\frac{1}{x}\right) = \frac{1}{1 - \frac{1}{\frac{1}{x}}} = \frac{\frac{1}{x}}{\frac{1}{x} - 1} \left[\because f_1(x) = \frac{1}{x}\right]$

$\Rightarrow (f_2 o J)(x) = \frac{x}{x-1} \left[\frac{1}{x} \text{ is replaced by } x\right]$

$\Rightarrow f_2(J(x)) = \frac{x}{x-1}$

$\Rightarrow 1 - J(x) = \frac{x}{x-1} [\because f_2(x) = 1 - x]$

$\therefore J(x) = 1 - \frac{x}{x-1} = \frac{1}{1-x} = f_3(x)$

Question138

Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then
[Online April 16, 2018]

Options:

A. Both R_1 and R_2 are transitive relations

B. Both R_1 and R_2 are symmetric relations

C. Range of R_2 is $\{1, 2, 3, 4\}$

D. Range of R_1 is {2, 4, 8}

Answer: C

Solution:

Solution:
Here, $R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$ and
$R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$
For R_1; $2x + y = 10$ and $x, y \in N$
So, possible values for x and y are:
$x = 1, y = 8$ i.e. (1,8);
$x = 2, y = 6$ i.e. (2,6);
$x = 3, y = 4$ i.e. (3,4) and $x = 4, y = 2$ i.e. (4,2).
$R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$
Therefore, Range of R_1 is $\{2, 4, 6, 8\}$
R_1 is not symmetric
Also, R_1 is not transitive because $(3, 4), (4, 2) \in R_1$ but $(3,2) \notin R_1$
Thus, options A, B and D are incorrect.
For R_2; $x + 2y = 10$ and $x, y \in N$
So, possible values for x and y are:$x = 8, y = 1$ i.e. (8,1);
$x = 6, y = 2$ i.e. (6,2);
$x = 4, y = 3$ i.e. (4,3) and
$x = 2, y = 4$ i.e. (2, 4)
$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$
Therefore, Range of R_2 is $\{1, 2, 3, 4\}$
R_2 is not symmetric and transitive.

Question139

Consider the following two binary relations on the set $A = \{a, b, c\}$: $R_1 = \{ (c, a)(b, b), (a, c), (c, c), (b, c), (a, a) \}$and $R_2 = \{ (a, b), (b, a), (c, c) ,(c, a), (a, a), (b, b), (a, c)$. Then [Online April 15, 2018]

Options:

A. R_2 is symmetric but it is not transitive

B. Both R_1 and R_2 are transitive

C. Both R_1 and R_2 are not symmetric

D. R_1 is not symmetric but it is transitive

Answer: A

Solution:

Solution:
Both R_1 and R_2 are symmetric as
For any $(x, y) \in R_1$, we have
$(y, x) \in R_1$ and similarly for R_2
Now, for R_2, $(b, a) \in R_2$, $(a, c) \in R_2$ but $(b, c) \notin R_2$
Similarly, for R_1, $(b, c) \in R_1$, $(c, a) \in R_1$ but $(b, a) \notin R_1$

Therefore, neither R_1 nor R_2 is transitive.

Question140

Let $f: A \to B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is
[Online April 15, 2018]

Options:

A. invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$

B. invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$

C. no invertible

D. invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

Answer: D

Solution:

Solution:
Suppose $y = f(x)$
$\Rightarrow y = \frac{x-1}{x-2}$
$\Rightarrow yx - 2y = x - 1$
$\Rightarrow (y-1)x = 2y - 1$
$\Rightarrow x = f^{-1}(y) = \frac{2y-1}{y-1}$
As the function is invertible on the given domain and its inverse can be obtained as above.

Question141

The function $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is:
[2017]

Options:

A. neither injective nor surjective

B. invertible

C. injective but not surjective

D. surjective but not injective

Answer: D

Solution:

Solution:

We have $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$,

$f(x) = \frac{x}{1+x^2} \; \forall x \in R$

$\Rightarrow f'(x) = \frac{(1+x^2).1 - x.2x}{(1+x^2)^2} = \frac{-(x+1)(x-1)}{(1+x^2)^2}$

sign of $f'(x)$

$\Rightarrow f'(x)$ changes sign in different intervals.

$\therefore$ Not injective Now $y = \frac{x}{1+x^2}$

$\Rightarrow y + yx^2 = x$

$\Rightarrow yx^2 - x + y = 0$

For $y \neq 0$, $D = 1 - 4y^2 \geq 0$

$\Rightarrow y \in \left[\frac{-1}{2}, \frac{1}{2}\right] - \{0\}$

For $y = 0 \Rightarrow x = 0$

$\therefore$ Range is $\left[\frac{-1}{2}, \frac{1}{2}\right]$

$\Rightarrow$ Surjective but not injective

Question142

The function $f: N \to N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, whereN is set of natural numbers and [x] denotes the greatest integer less than or equal to x, is:
[Online April 9, 2017]

Options:

A. one-one and onto.

B. one-one but not onto.

C. onto but not one-one.

D. neither one-one nor onto.

Answer: D

Solution:

Solution:

$\left.\begin{array}{l} f(1) = 1 - 5[1/5] = 1 \\ f(6) = 6 - 5[6/5] = 1 \end{array}\right\} \to$ Many one

$f(10) = 10–5(2) = 0$ which is not in co-domain.

Neither one–one nor onto.

Question143

Let $f(x) = 2^{10}.x + 1$ and $g(x) = 3^{10}.x - 1$. If $(f o g)(x) = x$, then x is equal to:
[Online April 8, 2017]

Options:

A. $\frac{3^{10}-1}{3^{10}-2^{-10}}$

B. $\frac{2^{10}-1}{2^{10}-3^{-10}}$

C. $\frac{1-3^{-10}}{2^{10}-3^{-10}}$

D. $\frac{1-2^{-10}}{3^{10}-2^{-10}}$

Answer: D

Solution:

Solution:

$f(g(x)) = x$

$\Rightarrow f(3^{10}x-1) = 2^{10}(3^{10}.x-1)+1 = x$

$\Rightarrow 2^{10}(3^{10}x-1)+1 = x$

$\Rightarrow x(6^{10}-1) = 2^{10}-1$

$\Rightarrow x = \frac{2^{10}-1}{6^{10}-1} = \frac{1-2^{-10}}{3^{10}-2^{-10}}$

Question144

For $x \in R$, $x \neq 0$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))n = 0, 1, 2, ...$ Then the value of $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to :

[Online April 9, 2016]

Options:

A. $\frac{8}{3}$

B. $\frac{4}{3}$

C. $\frac{5}{3}$

D. $\frac{1}{3}$

Answer: C

Solution:

Solution:

$f_1(x) = f_{0+1}(x) = f_0(f_0(x)) = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x}$

$f_2(x) = f_{1+1}(x) = f_0(f_1(x)) = \frac{1}{1-\frac{x-1}{x}} = x$

$$f_3(x) = f_{2+1}(x) = f_0(f_2(x)) = f_0(x) = \frac{1}{1-x}$$

$$f_4(x) = f_{3+1}(x) = f_0(f_3(x)) = \frac{x-1}{x}$$

$$\therefore f_0 = f_3 = f_6 = \ldots. = \frac{1}{1-x}$$

$$f_1 = f_4 = f_7 = \ldots\ldots = \frac{x-1}{x}$$

$$f_2 = f_5 = f_8 = \ldots\ldots.. = x$$

$$f_{100}(3) = \frac{3-1}{3} = \frac{2}{3} f_1\left(\frac{2}{3}\right) = \frac{\frac{2}{3}-1}{\frac{2}{3}} = -\frac{1}{2}$$

$$f_2\left(\frac{3}{2}\right) = \frac{3}{2}$$

$$\therefore f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right) = \frac{5}{3}$$

Question145

Let $A = \{x_1, x_2, \ldots.., x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \to B$ that are onto, if there exist exactly three elements x in A such that $f(x) = y_2$, is equal to

(Online April 11, 2015)

Options:

A. $14.{}^7C_3$

B. $16.{}^7C_3$

C. $14.{}^7C_2$

D. $12..{}^7C_2$

Answer: A

Solution:

Solution:

Number of onto function such that exactly three elements in $x \in A$ such that $f(x) = \frac{1}{2}$ is equal to

$= {}^7C_3, \{2^4 - 2\} = 14 \,.\, {}^7C_3$

Question146

Let $f : R \to R$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is:

[Online April 19, 2014]

Options:

A. both one-one and onto

B. one-one but not onto

C. onto but not one-one

D. neither one-one nor onto.

Answer: C

Solution:

Solution:

$f(x) = \frac{|x| - 1}{|x| + 1}$

for one-one function if $f(x_1) = f(x_2)$ then

x_1 must be equal to x_2

Let $f(x_1) = f(x_2)$

$\frac{|x_1| - 1}{|x_1| + 1} = \frac{|x_2| - 1}{|x_2| + 1}$ $|x_1||x_2| + |x_1| - |x_2| - 1 = |x_1||x_2| - |x_1| + |x_2| - 1$

$\Rightarrow |x_1| - |x_2| = |x_2| - |x_1|$

$2|x_1| = 2|x_2|$

$|x_1| = |x_2|$

$x_1 = x_2, x_1 = -x_2$

here x_1 has two values therefore function is many one function.

For onto : $f(x) = \frac{|x| - 1}{|x| + 1}$

for every value of $f(x)$ there is a value of x in domain set.

If $f(x)$ is negative then $x = 0$

for all positive value of $f(x)$, domain contain atleast one element. Hence $f(x)$ is onto function.

Question147

Let P be the relation defined on the set of all real numbers such that $P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$. Then P is:
[Online April 9, 2014]

Options:

A. reflexive and symmetric but not transitive.

B. reflexive and transitive but not symmetric.

C. symmetric and transitive but not reflexive.

D. an equivalence relation.

Answer: D

Solution:

Solution:

$P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$

For reflexive :

$\sec^2 a - \tan^2 a = 1$(true $\forall a$)

For symmetric :

$\sec^2 b - \tan^2 a = 1$

L.H.S

$1 + \tan^2 b - (\sec^2 a - 1) = 1 + \tan^2 b - \sec^2 a + 1$

$= -(\sec^2 a - \tan^2 b) + 2$

$= -1 + 2 = 1$
So, Relation is symmetric For transitive :
if $\sec^2 a - \tan^2 b = 1$ and $\sec^2 b - \tan^2 c = 1$
$\sec^2 a - \tan^2 c = (1 + \tan^2 b) - (\sec^2 b - 1)$
$= -\sec^2 b + \tan^2 b + 2$
$= -1 + 2 = 1$
So, Relation is transitive.
Hence, Relation P is an equivalence relation

Question148

Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$, where [n] denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} f(n)$ is equal to:
[Online April 19, 2014]

Options:

A. 56

B. 689

C. 1287

D. 1399

Answer: D

Solution:

Solution:
Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$
where [n] is greatest integer function,
$= \left[0.33 + \frac{3n}{100}\right]n$
For $n = 1, 2, \ldots, 22$, we get $f(n) = 0$ and for $n = 23, 24, \ldots, 55$, we get $f(n) = 1 \times n$ For $n = 56$, $f(n) = 2 \times n$
So, $\sum_{n=1}^{56} f(n) = 1(23) + 1(24) + \ldots + 1(55) + 2(56)$
$= (23 + 24 + \ldots + 55) + 112$
$= \frac{33}{2}[46 + 32] + 112$
$= \frac{33}{2}(78) + 112 = 1399$

Question149

Let f be an odd function defined on the set of real numbers such that for $x \geq 0$, $f(x) = 3\sin x + 4\cos x$ Then $f(x)$ at $x = -\frac{11\pi}{6}$ is equal to:
[Online April 11, 2014]

Options:

A. $\frac{3}{2} + 2\sqrt{3}$

B. $-\frac{3}{2}+2\sqrt{3}$

C. $\frac{3}{2}-2\sqrt{3}$

D. $-\frac{3}{2}-2\sqrt{3}$

Answer: C

Solution:

Solution:
Given f be an odd function
$f(x)=3\sin x+4\cos x$
Now, $f\left(\frac{-11\pi}{6}\right)=3\sin\left(\frac{-11\pi}{6}\right)+4\cos\left(\frac{-11\pi}{6}\right)$
$f\left(\frac{-11\pi}{6}\right)=3\sin\left(-2\pi+\frac{\pi}{6}\right)+4\cos\left(-2\pi+\frac{\pi}{6}\right)$
$f\left(\frac{-11\pi}{6}\right)=3\sin\left\{-\left(2\pi-\frac{\pi}{6}\right)\right\}+4\cos\left\{-\left(2\pi-\frac{\pi}{6}\right)\right\}$

$$\left\{\begin{array}{c}\text{For odd functions}\\ \sin(-\theta)=-\sin\theta\\ \text{and }\cos(-\theta)=\cos\theta\end{array}\right\}$$

$\therefore f\left(\frac{-11\pi}{6}\right)=-3\sin\left(2\pi-\frac{\pi}{6}\right)-4\cos\left(2\pi-\frac{\pi}{6}\right)$
$\Rightarrow f\left(\frac{-11\pi}{6}\right)=+3\sin\left(\frac{\pi}{6}\right)-4\cos\frac{\pi}{6}$
$\Rightarrow f\left(\frac{-11\pi}{6}\right)=3\times\frac{1}{2}-4\times\frac{\sqrt{3}}{2}$
or $f\left(\frac{-11\pi}{6}\right)=\frac{3}{2}-2\sqrt{3}$

Question150

If g is the inverse of a function f and $f'(x)=\frac{1}{1+x^5}$, then g′(x) is equal to: [2014]

Options:

A. $\frac{1}{1+\{g(x)\}^5}$

B. $1+\{g(x)\}^5$

C. $1+x^5$

D. $5x^4$

Answer: B

Solution:

Solution:
Since f(x) and g(x) are inverse of each other
$\therefore g'(f(x))=\frac{1}{f'(x)}$

$\Rightarrow g'(f(x)) = 1 + x^5 \left(\because f'(x) = \frac{1}{1+x^5}\right)$
Here $x = g(y)$
$\therefore g'(y) = 1 + [g(y)]^5$
$\Rightarrow g'(x) = 1 + (g(x))^5$

Question151

Let $R = \{(x, y) : x, y \in N$. and $x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is:
[Online April 23, 2013]

Options:

A. reflexive but neither symmetric nor transitive.

B. symmetric and transitive.

C. reflexive and symmetric,

D. reflexive and transitive.

Answer: D

Solution:

Solution:
$R = \{(x, y) : x, y \in N$ and $x^2 - 4xy + 3y^2 = 0\}$
Now, $x^2 - 4xy + 3y^2 = 0$
$\Rightarrow (x - y)(x - 3y) = 0$
$\therefore x = y$ or $x = 3y$
$\therefore R = \{(1, 1), (3, 1), (2, 2), (6, 2), (3, 3)(9, 3), \ldots\ldots\}$
Since (1, 1), (2, 2), (3, 3), ... are present in the relation, therefore R is reflexive.
Since (3,1) is an element of R but (1,3) is not the element of R, therefore R is not symmetric
Here $(3,1) \in R$ and $(1,1) \in R \Rightarrow (3, 1) \in R$ $(6,2) \in R$ and $(2,2) \in R \Rightarrow (6, 2) \in R$
For all such $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow (a, c) \in R$
Hence R is transitive.

Question152

Let $R = \{(3, 3)(5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5)\}$ be a relation on the set $A = \{3, 5, 9, 12\}$. Then, R is :
[Online April 22, 2013]

Options:

A. reflexive, symmetric but not transitive.

B. symmetric, transitive but not reflexive.

C. an equivalence relation.

D. reflexive, transitive but not symmetric.

Answer: D

Solution:

Solution:
Let $R = \{(3,3),(5,5),(9,9),(12,12),(5,12),(3,9),(3,12),(3,5)\}$ be a relation on set $A = \{3,5,9,12\}$
Clearly, every element of A is related to itself.
Therefore, it is a reflexive.
Now, R is not symmetry because 3 is related to 5 but 5 is not related to 3.
Also R is transitive relation because it satisfies the property that if aRb and bRc then aRc.

Question153

Let A = {1, 2, 3, 4} and R : A → A be the relation defined by R = {(1, 1), (2, 3), (3, 4), (4, 2)}. The correct statement is: [Online April 9, 2013]

Options:

A. R does not have an inverse.

B. R is not a one to one function.

C. R is an onto function.

D. R is not a function.

Answer: C

Solution:

Solution:
Domain $= \{1,2,3,4\}$
Range $= \{1,2,3,4\}$
∴ Domain = Range
Hence the relation R is onto function.

Question154

If P(S) denotes the set of all subsets of a given set S, then the number of one-to-one functions from the set S = {1, 2, 3} to the set P(S) is [Online May 19, 2012]

Options:

A. 24

B. 8

C. 336

D. 320

Answer: C

Solution:

Solution:
Let $S = \{1, 2, 3\} \rightarrow n(S) = 3$
Now, $P(S) =$ set of all subsets of S
total no. of subsets $= 2^3 = 8$
$\therefore n[P(S)] = 8$
Now, number of one-to-one functions from $S \rightarrow P(S)$ is ${}^8P_3 = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$

Question155

If $A = \{ x \in z^+ : x < 10$ and x is a multiple of 3 or 4 $\}$, where z^+ is the set of positive integers, then the total number of symmetric relations on A is.
[Online May 12, 2012]

Options:

A. 2^5

B. 2^{15}

C. 2^{10}

D. 2^{20}

Answer: B

Solution:

Solution:
A relation on a set A is said to be symmetric iff $(a, b) \text{ in } A \Rightarrow (b, a) \in A, \forall a, b \in A$
Here $A = \{3, 4, 6, 8, 9\}$
Number of order pairs of $A \times A = 5 \times 5 = 25$
Divide 25 order pairs of $A \text{ times } A$ in 3 parts as follows:
Part – A : (3, 3), (4, 4), (6, 6), (8, 8), (9, 9)
Part – B : (3, 4), (3, 6), (3, 8), (3, 9), (4, 6), (4, 8),(4, 9), (6, 8), (6, 9), (8, 9)
Part – C : (4, 3), (6, 3), (8, 3),(9, 3), (6, 4), (8, 4), (9, 4), (8, 6), (9, 6), (9, 8)
In part – A, both components of each order pair are same.
In part – B, both components are different but not two such order pairs are present in which first component of one order pair is the second component of another order pair and vice-versa.
In part–C, only reverse of the order pairs of part –B are present i.e., if (a, b) is present in part – B, then (b, a) will be present in part –C
For example (3, 4) is present in part – B and (4, 3) present in part –C.
Number of order pair in A, B and C are 5, 10 and 10 respectively.
In any symmetric relation on set A, if any order pair of part –B is present then its reverse order pair of part –C will must be also present.
Hence number of symmetric relation on set A is equal to the number of all relations on a set D, which contains all the order pairs of part –A and part– B.
Now $n(D) = n(A) + n(B) = 5 + 10 = 15$
Hence number of all relations on set $D = (2)^{15}$
$\Rightarrow$ Number of symmetric relations on set $D = (2)^{15}$

Question156

The range of the function $f(x) = \frac{x}{1+|x|}, x \in R$, is
[Online May 7, 2012]

Options:

A. R

B. (-1,1)

C. $R - \{0\}$

D. [-1,1]

Answer: B

Solution:

Solution:

$f(x) = \frac{x}{1+|x|}, x \in R$

If $x > 0, |x| = x \Rightarrow f(x) = \frac{x}{1+x}$

which is not defined for $x = -1$

If $x < 0, |x| = -x \Rightarrow f(x) = \frac{x}{1-x}$ which is not defined for $x = 1$

Thus $f(x)$ defined for all values of R except 1 and -1

Hence, range $= (-1, 1)$

Question157

Let A and B be non empty sets in R and f : A → B is a bijective function.
Statement 1: f is an onto function.
Statement 2: There exists a function g : B → A such that fog $= I_B$
[Online May 26, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

C. Statement 1 is false, Statement 2 is true.

D. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1.

Answer: D

Solution:

Solution:
Let A and B be non-empty sets in R.
Let $f : A \rightarrow B$ is bijective function.
Clearly statement - 1 is true which says that f is an onto function.
Statement -2 is also true statement but it is not the correct explanation for statement-1

Question158

Let R be the set of real numbers.
Statement-1: A = { (x, y) ∈ R × R : y − x is an integer } is an equivalence relation on R.
Statement- 2: B = { (x, y) ∈ R × R : x = αy for some rational number α } is an equivalence relation on R.
[2011]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:
$\because x - x = 0 \in I$ ($\therefore R$ is reflexive)
Let $(x, y) \in R$ as $x - y$ and $y - x \in I$ ($\because R$ is symmetric)
Now $x - y \in I$ and $y - z \in I \Rightarrow x - z \in I$
So, R is transative.
Hence R is equivalence.
Similarly as $x = \alpha y$ for $\alpha = 1$. B is reflexive symmetric and transative. Hence B is equivalence.
Both relations are equivalence but not the correct explanation.

Question159

The domain of the function f (x) = $\frac{1}{\sqrt{|x| - x}}$ **is**
[2011]

Options:

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(-\infty, \infty) - \{0\}$

D. $(-\infty, \infty)$

Answer: B

Solution:

Solution:

$f(x) = \frac{1}{\sqrt{|x| - x}}$, f (x) is define if $|x| - x > 0$

$\Rightarrow |x| > x, \Rightarrow x < 0$

Hence domain of f (x) is $(-\infty, 0)$

Question160

Let f be a function defined by
$f(x) = (x-1)^2 + 1, (x \geq 1)$
Statement -1: The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$
Statement -2: f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$
[2011 RS]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1.

C. Statement-1 is true, Statement-2 is false.

D. Statement-1 is false, Statement-2 is true.

Answer: A

Solution:

Solution:

Given f is a bijective function

$\therefore f : [1, \infty) \to [1, \infty)$

$f(x) = (x-1)^2 + 1, x \geq 1$

Let $y = (x-1)^2 + 1 \Rightarrow (x-1)^2 = y - 1$

$\Rightarrow x = 1 \pm \sqrt{y-1} \Rightarrow f^{-1}(y) = 1 \pm \sqrt{y-1}$

$\Rightarrow f^{-1}(x) = 1 + \sqrt{x-1} \{\therefore x \geq 1\}$

Hence statement- 2 is correct

Now $f(x) = f^{-1}(x)$

$\Rightarrow f(x) = x \Rightarrow (x-1)^2 + 1 = x$

$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

Hence statement- 1 is correct

Question161

Consider the following relations:
R = { (x, y) | x, y are real numbers and x = wy for some rational number w }; $S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \middle| m, n, p. \text{ and } q \text{ areintegers such that } n, q \neq 0 \text{ and } qm = pn \right\}$
. Then
[2010]

Options:

A. Neither R nor S is an equivalence relation

B. S is an equivalence relation but R is not an equivalence relation

C. R and S both are equivalence relations

D. R is an equivalence relation but S is not an equivalence relation

Answer: B

Solution:

Solution:
Let xRy.
$\Rightarrow x = wy \Rightarrow y = \frac{x}{w}$
$\Rightarrow (y, x) \notin R$
R is not symmetric
Let S : $\frac{m}{n} S \frac{p}{q}$
$\Rightarrow qm = pn \Rightarrow \frac{p}{q} = \frac{m}{n}$
$\because \frac{m}{n} = \frac{m}{n} \therefore$ reflexive
$\frac{m}{n} = \frac{p}{q} \Rightarrow \frac{p}{q} = \frac{m}{n} \therefore$ symmetric
Let $\frac{m}{n} S \frac{p}{q}, \frac{p}{q} S \frac{r}{s}$
$\Rightarrow qm = pn, ps = rq$
$\Rightarrow \frac{p}{q} = \frac{m}{n} = \frac{r}{s}$
$\Rightarrow ms = rn$ transitive
.S is an equivalence relation.

Question162

Let $f(x) = (x + 1)^2 - 1, x \geq -1$
Statement -1: The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}$.
Statement- 2: f is a bijection.
[2009]

Options:

A. Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

Answer: D

Solution:

Solution:
Given that $f(x) = (x + 1)^2 - 1, x \geq -1$

Clearly $D_f = [-1, \infty)$ but co-domain is not given. Therefore $f(x)$ is onto.
Let $f(x_1) = f(x_2)$
$\Rightarrow (x_1 + 1)^2 - 1 = (x_2 + 1)^2 - 1$
$\Rightarrow x_1 = x_2$
$\therefore f(x)$ is one-one, hence $f(x)$ is bijection
$\because (x + 1)$ being something $+ve$, $\forall x > -1$
$\therefore f^{-1}(x)$ will exist. Let $(x + 1)^2 - 1 = y$
$\Rightarrow x + 1 = \sqrt{y + 1}$ (+ve square root as $x + 1 \geq 0$)
$\Rightarrow x = -1 + \sqrt{y + 1}$
$\Rightarrow f^{-1}(x) = \sqrt{x + 1} - 1$
Then $f(x) = f^{-1}(x)$
$\Rightarrow (x + 1)^2 - 1 = \sqrt{x + 1} - 1$
$\Rightarrow (x + 1)^2 = \sqrt{x + 1} \Rightarrow (x + 1)^4 = (x + 1)$
$\Rightarrow (x + 1)[(x + 1)^3 - 1] = 0 \Rightarrow x = -1, 0$
$\therefore$ The statement- 1 and statement- 2 both are true.

Question163

Let R be the real line. Consider the following subsets of the plane R × R :
$S = \{ (x, y) : y = x + 1 \text{ and } 0 < x < 2 \}$
$T = \{ (x, y) : x - y \text{ is an integer } \}$
Which one of the following is true?
[2008]

Options:

A. Neither S nor T is an equivalence relation on R

B. Both S and T are equivalence relation on R

C. S is an equivalence relation on R but T is not

D. T is an equivalence relation on R but S is not

Answer: D

Solution:

Solution:
Given that
$S = \{ (x, y) : y = x + 1$ and $0 < x < 2 \}$
$\because x \neq x + 1$ for any $x \in (0, 2)$
$\Rightarrow (x, x) \notin S$
So, S is not reflexive.
Hence, S in not an equivalence relation.
Given $T = \{x, y) : x - y$ is an integer $\}$
$\because x - x = 0$ is an integer, $\forall x \in R$
So, T is reflexive.
Let $(x, y) \in T \Rightarrow x - y$ is an integer then $y - x$ is also an integer $\Rightarrow (y, x) \in R$
$\therefore T$ is symmetric
If $x - y$ is an integer and $y - z$ is an integer then $(x - y) + (y - z) = x - z$ is also an integer.
$\therefore T$ is transitive
Hence T is an equivalence relation.

Question164

Let f : N → Y be a function defined as f (x) = 4x + 3 where Y = { y ∈ N : y = 4x + 3 for some x ∈ N }.
Show that f is invertible and its inverse is
[2008]

Options:

A. $g(y) = \frac{3y+4}{3}$

B. $g(y) = 4 + \frac{y+3}{4}$

C. $g(y) = \frac{y+3}{4}$

D. $g(y) = \frac{y-3}{4}$

Answer: D

Solution:

Solution:
Clearly $f(x) = 4x + 3$ is one one and onto, so it is invertible.
Let $f(x) = 4x + 3 = y$
$\Rightarrow x = \frac{y-3}{4}$
$\therefore g(y) = \frac{y-3}{4}$

Question165

Let W denote the words in the English dictionary. Definethe relation R by R = (x, y) ∈ W × W | the words x and yhave at least one letter in common.} Then R is
[2006]

Options:

A. not reflexive, symmetric and transitive

B. relexive, symmetric and not transitive

C. reflexive, symmetric and transitive

D. reflexive, not symmetric and transitive

Answer: B

Solution:

Solution:
Clearly $(x, x) \in R, \forall x \in W$
∵ All letter are common in some word. So R is reflexive.
Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have at least one letter in common. So, R is symmetric.
But R is not transitive for example
Let x = BOY , y = TOY and z = THREE

then $(x, y) \in R$(O, Y are common) and $(y, z) \in R$(T is common) but $(x, z) \notin R$. (as no letter is common)

Question166

A real valued function $f(x)$ satisfies the functional equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ where a is a given constant and $f(0) = 0$, $f(2a-x)$ is equal to [2005]

Options:

A. $-f(x)$

B. $f(x)$

C. $f(a) + f(a-x)$

D. $f(-x)$

Answer: A

Solution:

Solution:
Given that $f(0) = 0$ and put
$x = 0, y = 0$
$f(0) = f^2(0) - f^2(a)$
$\Rightarrow f^2(a) = 0 \Rightarrow f(a) = 0$
$f(2a-x) = f(a-(x-a))$
$= f(a)f(x-a) - f(0)f(x)$
$= f(a)f(x-a) - f(x) = -f(x)$
$\Rightarrow f(2a-x) = -f(x)$

Question167

Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9),(3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. The relation is [2005]

Options:

A. reflexive and transitive only

B. reflexive only

C. an equivalence relation

D. reflexive and symmetric only

Answer: A

Solution:

Solution:
R is reflexive and transitive only.
Here(3, 3), (6, 6), (9, 9), (12, 12) $\in$ R [So, reflexive]
(3, 6), (6, 12), (3, 12) $\in$ R[So, transitive]
$\because$(3, 6) $\in$ R but (6,3)$\notin$R[So, non-symmetric]

Question168

Let $f : (-1, 1) \rightarrow B$, be a function defined by$f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one − one and onto when B is the interval
[2005]

Options:

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: D

Solution:

Solution:
Given $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$
for $x \in (-1, 1)$
If $x \in (-1, 1) \Rightarrow \tan^{-1}x \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$
$\Rightarrow 2\tan^{-1}x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
Clearly, range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
For f to be onto, codomain = range
$\therefore$ Co-domain of function $= B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Question169

The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then
[2004]

Options:

A. $f(x) = -f(-x)$

B. $f(2+x) = f(2-x)$

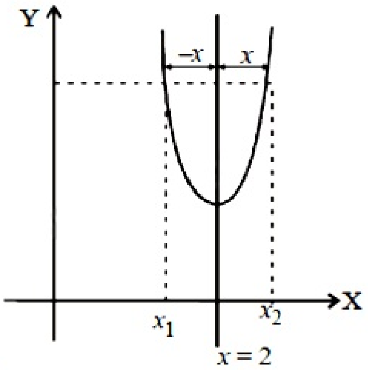
C. f (x) = f (−x)

D. f (x + 2) = f (x − 2)

Answer: B

Solution:

Solution:
(b) Given that a graph symmetrical. with respect to line $x = 2$ as shown in the figure.



From the figure
$f(x_1) = f(x_2)$, where $x_1 = 2 - x$ and $x_2 = 2 + x$
$\therefore f(2 - x) = f(2 + x)$

Question170

Let R = {(1,3),(4, 2), (2, 4),(2,3),(3,1)} be a relation on the set A = {1, 2,3, 4}.. The relation R is
[2004]

Options:

A. reflexive

B. transitive

C. not symmetric

D. a function

Answer: C

Solution:

Solution:
∵(1, 1) ∉ R ⇒ R is not reflexive
∵(2, 3) ∈ R but (3,2)∉R
∴R is not symmetric
∵(4, 2) and (2,4)∈R but (4,4)∉R
⇒R is not transitive

Question171

If $f : R \rightarrow S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is
[2004]

Options:

A. [-1,3]

B. [-1,1]

C. [0,1]

D. [0,3]

Answer: A

Solution:

Solution:
Given that f (x) is onto
$\therefore$ range of f (x) = codomain = S
Now, $f(x) = \sin x - \sqrt{3}\cos x + 1$
$= 2\sin\left(x - \frac{\pi}{3}\right) + 1$
we know that $-1 \le \sin\left(x - \frac{\pi}{3}\right) \le 1$
$-1 \le 2\sin\left(x - \frac{\pi}{3}\right) + 1 \le 3 \therefore f(x) \in [-1, 3] = S$

Question172

Domain of definition of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is
[2003]

Options:

A. $(-1,0) \cup (1, 2) \cup (2, \infty)$

B. $(a, 2)$

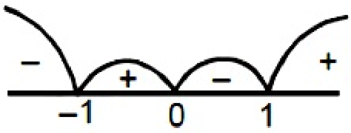
C. $(-1,0) \cup (a, 2)$

D. $(1,2) \cup (2, \infty)$

Answer: A

Solution:

Solution:
$f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$
$4 - x^2 \neq 0; x^3 - x > 0$
$x \neq \pm\sqrt{4}$ and $-1 < x < 0$ or $1 < x < \infty$



$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\}$
$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$

Question173

If f : R → R satisfies $f(x + y) = f(x) + f(y)$, for all x, $y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is
[2003]

Options:

A. $\frac{7n(n + 1)}{2}$

B. $\frac{7n}{2}$

C. $\frac{7(n + 1)}{2}$

D. $7n + (n + 1)$

Answer: A

Solution:

Solution:
$f(x + y) = f(x) + f(y)$
$\because f(1) = 7$
$f(2) = f(1 + 1) = f(1) + f(1) = 14$
$f(3) = f(1 + 2) = f(1) + f(2) = 21$
– – – – – – – – – – – –
– – – – – – – – – – – –
$\therefore \sum_{r=1}^{n} f(r) = 7(1 + 2 + 3 ..... + n)$
$= \frac{7n(n + 1)}{2}$

Question174

A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, \text{when n is odd} \\ -\frac{n}{2}, \text{when n is even} \end{cases} \text{ is}$$

[2003]

Options:

A. neither one -one nor onto

B. one-one but not onto

C. onto but not one-one

D. one-one and onto both.

Answer: D

Solution:

Solution:
We have $f : N \to I$
Let x and y are two even natural numbers, and $f(x) = f(y) \Rightarrow \frac{-x}{2} = \frac{-y}{2} \Rightarrow x = y$
$\therefore f(n)$ is one-one for even natural number.
Let x and y are two odd natural numbers and $f(x) = f(y) \Rightarrow \frac{x-1}{2} = \frac{y-1}{2} \Rightarrow x = y$
$\therefore f(n)$ is one-one for odd natural number.
Hence f is one-one.
Let $y = \frac{n-1}{2} \Rightarrow 2y + 1 = n$
This shows that n is always odd number for $y \in I$ ......(i)
and $y = \frac{-n}{2} \Rightarrow -2y = n$
This shows that n is always even number for $y \in I$ ......(ii)
From (i) and (ii)
Range of $f = I =$ codomain
$\therefore f$ is onto.
Hence f is one one and onto both.
